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A New Search-and-Matching Computable General Equilibrium Model: Progressive Consumption Taxation and Unemployment Equilibrium Effects on Growth, Unemployment, and Incidence

by

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ABSTRACT

A New Search-and-Matching Computable General Equilibrium Model: Progressive Consumption Taxation and Unemployment Equilibrium Effects on Growth, Unemployment, and Incidence

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I develop a model that includes a search-and-matching model of unemployment equilibrium with endogenous wage bargaining in the labor market to evaluate consumption tax reforms in the U.S.. I also improve such framework to examine consumption tax reforms with progressive components –a Hall-Rabushka style flat tax and the Bradford X-tax– and to measure their impact on different income groups instead of a single representative individual.

My model simulates the long-term effects (from a benchmark to a steady state equilibrium) of reforms with income-heterogeneous individuals, in a closed economy. This study contributes to the literature by accounting simultaneously for the unemployment equilibrium effects and income heterogeneity in a consumption-based reform; and by doing so, it provides a more robust base for the analysis of the efficiency and incidence of tax reforms. I use the model to test if growth, capital accumulation and economic efficiency that typically occur under standard CGE analyses of consumption tax reforms are enhanced or reduced with the addition of a search-and-matching labor market framework. I also analyze if such reforms are distributionally neutral or if they affect income groups disproportionately. Finally, by evaluating two consumption tax reforms with different progressive tax structures –which include only a standard deduction (the flat tax) or progressive marginal tax rates (the X-tax)– I analyze the impact of such differences in the results.

Both, the flat tax and the X-tax, lead to an average reduction in after-tax bargained wages. However, they also leave firms in better conditions to create more jobs by opening more vacancies, which consequently, also lead to a reduction in the level of unemployment. Therefore, the inclusion of unemployment equilibrium counters in some degree the decrease in the aggregate individual labor supply, caused by the reforms' implementation, by reducing the unemployment level and increasing the total employed labor.

The X-tax, which keeps roughly the same average income tax due to its progressive tax schedule, has a lower impact on lower and middle-income groups than the flat tax. The last increases the average income tax for all income groups except the top decile, reducing wages more than the X-tax. Nevertheless, in both scenarios, it is the top two percent who obtains the largest increase in welfare, measured by its equivalent variation.

Contrary to standard results, capital drops. Vacancies, now considered by firms as investment projects that increase their effective labor, compete with capital for resources, thus reducing the level of capital accumulation compared with a standard CGE model with a competitive labor market. Also, since the model only analyzes the state-state equilibrium, it misses the gains from reforms on old capital, besides the new investment expensing. Capital has the largest drop with the X-tax, which has an increase in the business income tax rate of five percent points to match the highest individual income tax rate. Capital reduces less that the flat tax because it also has a reduction in the business tax rate of four percent points. Additionally, output only increases under the X-tax, because the unemployment reduction does not compensate the large drop in the aggregate individual labor under the flat tax.

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To my parents, Víctor and Mercedes.

Chapter 1

Search-and-Matching CGE Modelling To Analyze Consumption Tax Reforms: Unemployment Equilibrium Effects on Efficiency and Incidence

1.1 Introduction

The advantages (and disadvantages) of replacing the current U.S. national tax system with a consumption-based tax has been widely discussed, and computable general equilibrium (CGE) models have been a useful tool for its analysis.[1] CGE estimates from consumption-based tax reforms for the U.S. have shown considerable efficiency gains in output and capital accumulation, especially in the long run. In particular, proportional consumption tax reforms delivered significant increases in national income and capital stock (9.5 and 25.4 percent respectively (Altig et al. 2001)).

Transitional effects and neutrality of horizontal and vertical incidence are factors that reduce, eliminate (or potentially could even reverse) the welfare increase achieved by the efficiency gains in output and capital accumulation (Altig et al. 2001). Reform's transitional effects include changes in goods' prices, wages, the value of debt and equity, and a one-time windfall loss on previously owned capital (Bradford 1996). Its incidence might affect the elderly (Bradford 1996), and lower income families and middle class (Mieszkowski

[1] A computable general equilibrium is a model that uses numerical methods to find its equilibrium. It is widely used to analyze the effects changes in public policy variables (taxes, for instance) that affect different markets simultaneously, and for which there is not enough data to perform a controlled experiment. For a compendium of previous work and recent developments in CGE modeling see Dixon and Jorgenson (2013a, 2013b).

and Palumbo 2002) disproportionally. For example, some proposals include a relief to previously owned capital, allowing to depreciate it temporarily following the previous rule. Others include progressive components such as a standard deduction (Hall-Rabushka flat tax (Hall and Rabushka 1983; Hall and Rabushka 2007)) or a progressive tax rate structure (X tax (Bradford 1986; Bradford 2005)). Estimates have shown a considerable reduction in the gains achieved by those reforms. For example, the X tax –a reform close to neutral along the income distribution– reaches lower output and capital stock gains than a proportional tax (3.1 and 4.4 percent less, respectively (Altig et al. 2001)). An additional increase to finance a transitional relief might not only significantly reduce the efficiency gains of the X-tax but also reverse its progressivity (Zodrow 2007, 60). The inclusion of a transitional relief for previously owned capital in the flat tax reduces its gains roughly by half; its national income reduces from 4.5 to 1.9 percent and its capital stock gains from 15 to 8.3 percent (Altig et al. 2001). Additionally, in the long run, the flat tax's standard deduction only benefits the lowest and highest two percent of the population when it includes the transitional relief; the rest of the population is slightly worse off. Thus, larger efficiency gains come at the expense of welfare losses when transitional effects and incidence neutrality are accounted. That could diminish considerably the incentive to change to a consumption tax base (Gravelle 2002, 26).

Nonetheless, these results are based on the assumption that the labor market reaches a competitive equilibrium; wage and work hours supplied are optimal, and therefore labor market efficiency and incidence is measured including changes in wages and amount of hours destined to work away from the competitive equilibrium. Once this assumption is dropped, the efficiency and incidence results of a consumption-based tax reform could markedly change. The existence of unemployment in equilibrium implies that the equilibrium allocation is not a first-best market-clearing allocation anymore. Thus, public policy instruments that were considered distortionary in the context of a competitive labor market can now improve the welfare of individuals.

Under unemployment equilibrium, efficiency losses in the labor market due to a reduction in the hours supplied or the inclusion of progressive tax components have to be balanced with efficiency gains obtained from unemployment reduction. Similarly, in the context of unemployment equilibrium, unemployment insurance plays a significant role in the decision of work hours supplied.

Regarding efficiency analysis, the inclusion of unemployment equilibrium moderates the labor substitution caused by the reform implementation and enhanced by the inclusion of progressive components, in the long run. In this scenario, despite that a reduction in after-tax wages (due to the reform) might drop the individuals' supply of working hours, it also pressures wages downwards. Thus, it lowers the labor costs and increases firms' marginal rent from labor, which incentives the opening of job vacancies, and leads to a reduction in the unemployment rate. By the same logic, this model also moderates the effects of progressive components, which enhance the labor substitution effect caused by the reform.Additionally, the inclusion of unemployment insurance in the model is also vital. While it reduces the impact of unemployment on individuals' consumption and savings (by reducing the variability of their budget), it also reduces individuals' labor participation and job search effort (Tatsiramos and van Ours 2012); hence, it provides more leverage to people to bargain an increase in their wages. If the unemployment insurance is large enough, the increase in wages and consequent reduction in firms' marginal rent could offset any positive effect of a progressive reform on unemployment. It reduces the individuals' incentives to provide labor due to the increase in their non-wage income but also reduces the regressive incidence of consumption taxes on lower and middle-income groups.

Concerning incidence analysis, the inclusion of income heterogeneity accounts for idiosyncratic effects on wages for different income groups in a context of steady-state unemployment equilibrium. Lower income groups have a larger incentive to change their unemployment status; thus they have a lower leverage to bargain their wages. As a result, any pressure to reduce wages will have a larger impact on lower income groups than high-income ones. Therefore, if the effect of the drop in wages is large, the positive effect of progressive tax rates and consequent unemployment reduction on lower-income groups (explained in the previous paragraph) could shrink. The inclusion of unemployment insurance also affects individual's welfare under a consumption tax reform. In this case, the unemployment insurance benefits lower income groups more than high-income ones, by increasing their leverage to obtain larger wages and reducing the wage-reduction effect. The result on the welfare of each income group depends on which effect predominates. The aggregate effect on unemployment, growth, and consumption would depend on the net effect on each income group. Different income groups will be affected differently by tax policies, facing different potential gains from a job match; thus they will be subject to different bargained wages, affecting the overall efficiency and incidence of the reform.

Previous CGE models had not yet analyzed the effects of consumption-based reforms in the U.S. considering unemployment equilibrium in their modeling, although a departure from the assumption of a competitive labor market is essential. The primary goal of this study is to include a search-and-matching model of unemployment equilibrium with endogenous wage bargaining in the labor market to evaluate U.S. consumption tax reforms.[2]

[2]In unemployment equilibrium models with *efficiency wages* (Shapiro and Stiglitz 1984), firms offer wages larger than the competitive market ones to infuse higher levels of effort in workers; costs increase as a consequence of higher wages reducing the number of employees and producing unemployment in equilibrium. In the case of *collective bargaining* models (McDonald and Solow 1981), wages are bargained between the union and the firm, and workers decide between the wage offered by the union or the possible alternative outside the sector; the larger wages obtained from the rents shared during the bargaining also produce unemployment in the equilibrium.

Also, improve such framework to examine consumption tax reforms with progressive components –a Hall-Rabushka style flat tax and the Bradford X-tax– and to measure their impact on different income groups instead of a single representative individual.

It simulates the long-term efficiency effects and incidence of the reform (from a benchmark to a steady state equilibrium), in a closed economy with income-heterogeneous individuals. The use of a search-and-matching equilibrium responds to two important framework requirements for the object of this study; one is the need to incorporate the effects of unemployment in the tax reform endogenously; the other is the allowance of income heterogeneity in the process of wage determination. In the search-and-matching framework I use, unemployment is the result of the existence of labor market *frictions*; finding the appropriate match for a job is costly to firms, it takes time, and those matches last for a finite lapse. Thus, there is always a fraction of the population unemployed at any time as a consequence of those frictions. Also, the wage agreed is the result of the bargaining of firms and individuals (Pissarides 2000) on how to split the potential gains obtained from the job match; individuals obtain gains from getting a job, and firms obtain profits out of filling its vacant position. Income heterogeneity is reflected in the value set on employment by individuals and hence, also influences the wage bargaining process. By simultaneously accounting for the unemployment equilibrium effects and income heterogeneity in a consumption-based reform, this study provides a more robust base for the analysis of tax reforms' efficiency and incidence.

The ambiguity in the overall efficiency and incidence effects of a consumption-based reform in unemployment equilibrium points out two important aspects of labor market modeling: the complementarity between tax reform and unemployment insurance policies (Shimer and Smith 2001; Lehmann and Van der Linden 2007); and the necessity of an accurate measurement of individual labor supply responses (Gravelle 2002). In other words, a consumption-based reform could achieve a large degree of progressivity and efficiency, but depending on a certain level of unemployment benefits and individuals' labor supply responses (Sørensen 1999). My model accounts for the complementarity of tax reforms and unemployment insurance by modeling individual's decisions on consumption and savings assuming they are income heterogeneous and introducing bargained wages and unemployment in equilibrium. It also measures of individual labor supply responses more accurately by using individuals' utility preferences that allow for lower individual labor supply elasticities, which are more common in the empirical literature. Lower labor supply elasticities have a lower effect on output; hence using large elasticities would overstate the behavioral effect of the reform (Gravelle 2002). Since my model also takes into account the effect of the extensive margin of labor, employment and output effects in the long-run can still be large even under the assumption of low labor supply elasticities. In general, if the combination of positive efficiency effects of progressive consumption-based reforms, and the moderation of efficiency losses due to the substitution of labor for leisure are significant, the long-run benefits of a consumption tax reform might compensate for transitional concerns; while supporting the use (Koskela and Vilmunen 1996; Pissarides 1998), and optimality of progressive taxation (Lehmann, Parmentier, and Van Der Linden 2011). Given the existence of opposite forces determining the level of wages and unemployment, and the complexity of the model, the result is not clear without a simulation of the reform.

In this chapter, I discuss the framework for the development of the new model. First, I describe the search-and-matching unemployment equilibrium model. Next, I explain the core of proposals of consumption-based reforms, including some of which I apply in the second chapter. Finally, I provide insight on how the inclusion of this type of unemployment equilibrium changes the results of consumption-based reform analysis. The concluding section summarizes, mentions some caveats in the modeling, and proposes paths of future research.

1.2 Search-and-Matching General Equilibrium

Search-and-matching models have its precedent in one-sided job search modeling (McCall 1970), which is concerned with individuals' labor supply decision based on their expected gains from accepting a job offer or continue unemployed. In a search-and-matching set-up, firms' decision process is also taken into account. Hence, a search-and-matching framework adopts a general equilibrium approach, in which the objective of unemployed individuals and firms with job vacancies is also to find a good job match (Pissarides 2010), and expected gains are split in their bargained wage (Diamond 1982; Mortensen 1982). In that context, labor market *frictions* make the matching objective more difficult, producing a rough transition towards employment. Decisions occur in a decentralized labor market; individuals and firms internalize the unemployment and wage-bargaining mechanism to choose their labor demand and supply accordingly.

1.2.1 Job Vacancies and Unemployment Equilibrium

The population that participates in the labor market is either employed or unemployed, and they will either change their employment status or keep it. The population that becomes unemployed, the employment outflow (or unemployment inflow), is determined, in my model, exogenously.[3] The employment outflow is caused by productivity shocks that break-up a fraction of previously matched jobs. The percentage of employed individuals that become unemployed is known as the *separation rate*. The employment outflow is

[3] See Pissarides (2000) for endogenous employment outflow, and *on the job* employment search.

an important characteristic of the labor market because it assures the existence of unemployment in steady state equilibrium; without a positive separation rate, despite there are individuals left unmatched in every period, everyone would be eventually hired in the long term, and therefore, unemployment would not exist.

The population that becomes employed, the employment inflow (or unemployment outflow), is modeled using the *matching function*. The matching function, similar to a production function, represents the technology that determines the rate at which job matches occur, known as *matching rate*, based on combinations of unemployment and vacancy rates as inputs, and parameters that introduce the effects of labor market frictions in the outcome (Pissarides 2000, 6). *Frictions* is the jargon that refers to job search and time costs, market congestion, availability of virtual or physical spaces that facilitate the labor market exchange, and compatibility between job candidates and vacancies. These frictions reflect the fact that finding a good match is a costly endeavor. Even if the number of firms posting a vacancy is smaller than the number of unemployed individuals, and vice-versa, not all vacancies are necessarily filled, or unemployed individuals employed at a given period. In the matching function I use, there are two friction parameters.[4] One measures the job market's *matching efficiency*, such as the extent of availability of virtual or physical spaces that facilitate the labor exchange, or the compatibility between the skills offered by individuals and the ones demanded by the firms. The more efficient the market is, the larger is the matching rate. The other friction parameter measures the sensitivity of the matching rate to positive (and negative) externalities created by a relative increase in the number of firm vacancies or unemployed individuals (Petronglo and Pissarides 2001). A larger number of vacancies reduces the competition for the same position among individuals, but it also increases the competition among firms to fill those vacancies. In other words, it measures the

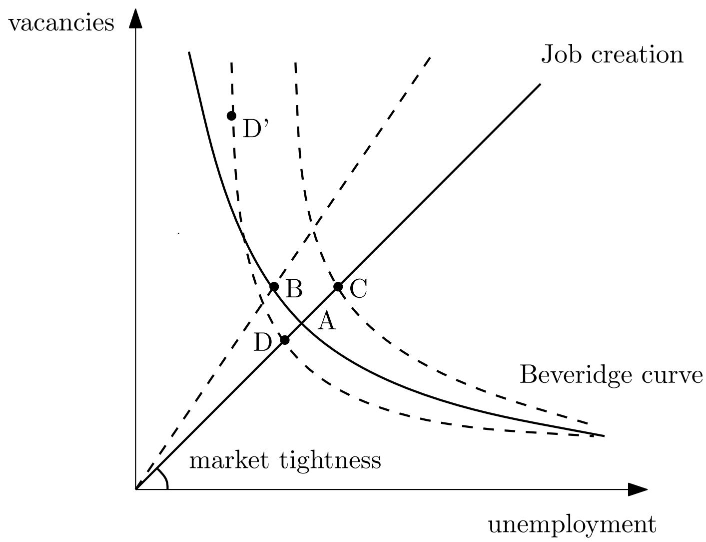
[4] I am using a Cobb-Douglas matching function.

sensitivity to market congestion, and the net effect on the matching rate depends on which congestion effect predominates. For example, if the congestion for firms is high (low for individuals), but the sensitivity towards unemployment's externality is low (high towards vacancies), then firms' high congestion outweighs the reduced individual's congestion.

A simple way to represent this market congestion is using the vacancy to unemployment ratio, known as *market tightness*. It reflects the congestion, or *tightness* that firms and individuals face when the number of job vacancies relative to the number of unemployed workers increases or decreases. A high vacancy to unemployment ratio, that is a high market tightness, increases firms mutual congestion to choose their best match but reduces the congestion of individuals who have more vacancy availability in relative terms. The opposite happens when the vacancy to unemployment ratio is low. The market tightness also embeds the information about the inputs of the matching function. Unemployment is determined endogenously in equilibrium; however, vacancy rates are affected by firms' labor demand decisions. Firm's decision on the equilibrium vacancy rate level is a crucial determinant of the market tightness, the matching rate, and equilibrium unemployment. To determine their vacancy requirement firms need to know the labor force they will count with for their production process; their decision relies on a vacancy rate adjustment mechanism to the equilibrium employed (hence, unemployed) population. In other words, firms decide their optimal market tightness. Firms choose their optimal market tightness by opening vacancies until the point where the marginal profit they could obtain by filling them is equal to the costs of opening them and keeping them open until they get filled. The steady state equilibrium is summarized in the *job creation equation*, represented in figures 1.1 and 1.2.

The steady state equilibrium between employment inflow and outflow –when they are offset– is summarized in the *Beveridge equation*, represented in figure 1.1. The Beveridge equation, along with the job creation equation determines the equilibrium vacancy and unemployment rates (figure 1.1). The equilibrium of vacancies and unemployment is a combination of the optimal vacancy decisions by the firm, and the labor market structure which include the market frictions. A useful way of think about it is that firms decide their optimal vacancy rate by making benefits equal to the costs, considering the fact that they will not be able to fill all of them. That is, firms internalize the matching process of the labor market. Vacancy rates and employment rates of equilibrium are the result, and hence unemployment equilibrium as well. For example, in figure 1.1, an increase in the separation rate would shift the Beveridge curve to the right, which also increases the number of vacancies available (firms need to cover those positions that were left open), and structural unemployment, given that some individuals became unemployed. A decrease in the market efficiency would have the same effect. Both results are represented by a movement from point A to C. Changes in the sensitivity to market congestion -to either unemployment or vacancy rates- would modify the shape of the Beveridge curve. The congestion effect that prevails, for firms or individuals, depends on the market tightness, which is the slope of the job creation curve. For example, in equilibrium D, after a decrease in the sensitivity towards unemployment, the lower congestion for firms produced by the lower vacancy rate outweighs the increased congestion for unemployed individuals who need to compete for fewer vacancies available. Thus, the level of unemployment drops. The opposite occurs in equilibrium D', where the vacancy rate is so high that the increased congestion for firms outweighs the lower congestion for individuals, increasing unemployment. On the job creation curve, a reduction in the average costs of job-fitting candidate's search would increase the number of new job openings or the time the vacancy is kept open; consequently, the matching rate increases and unemployment drops. This movement in the job creation curve is represented by a movement from point A to B in figure 1.1.

Figure 1.1 : Equilibrium vacancies and unemployment.



Individuals also maximize their utility contingent on being employed or unemployed; if employed they choose optimal hours of labor supply. The aggregate labor supply represents the total number of hours supplied in the economy in each a period, and from which labor market mechanism will employ a certain fraction in equilibrium, which will become the effectively employed labor force, later used by firms in their productive process. If individuals' labor supply decreases in the aggregate, firms will try to increase their market tightness by opening vacancies to keep the same level of profit (a movement from point A to B in figure 1.1), which leads to a reduction in the unemployment rate. Nevertheless, the most significant influence of individuals and their income heterogeneity is accounted in the wage determination process.

1.2.2 Bargaining and Wage Determination

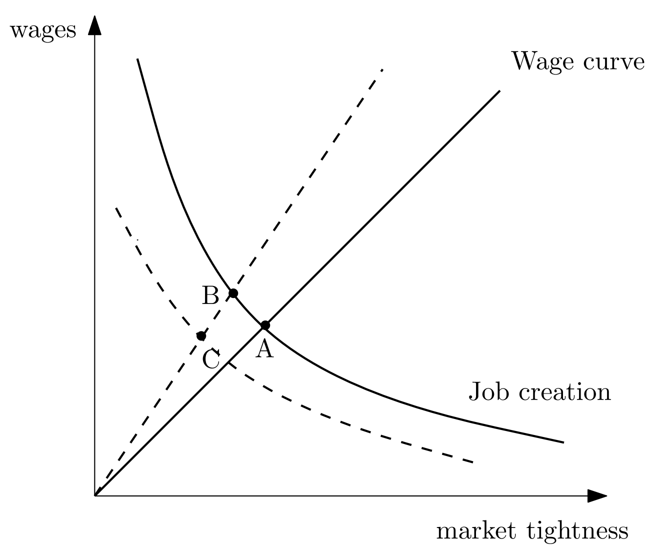
The wage determination process has its origin in bargaining theory (Nash 1950; Binmore, Rubinstein, and Wolinsky 1986; Shaked and Sutton 1984). The idea behind the wage setting is that the labor exchange produces a surplus, and the wage set determines how it will be split between employees and firms.

Labor market frictions create a monopolistic power during the job matching (Pissarides 2000) allowing firms and individuals to bargain for a larger share of the surplus. The wage agreed, represented in the *wage equation* (figure 1.2), maximizes the joint gains of firms and individuals extracted from their job agreement, weighted by their respective power, the *Nash bargaining solution*. In equilibrium, the marginal benefits and loses, either to individuals and firms, from a change in wages are offset.

To determine the gains, each participant determines the value they extract if they reach and agreement and when they do not. Based on the contingent benefits (or costs) they obtain under either employment or unemployment or having a vacancy filled or not, individuals and firms calculate the value of being in each state. For individuals, the value is the expected value obtained in their lifetime, considering that in each period they can be either employed or unemployed. The value of being employed represents the current benefit (utility) extracted from being employed and the discounted value of either being employed or unemployed the next period. Similarly for the unemployment state. In the case of firms, the value is the expected value obtained while economy produces, which is infinite, considering that vacancies can be filled or not. The differences between the value of being employed and being unemployed, and between the value of filling a vacancy or not, which are the gains from changing to one status to the other, represent the surplus produced by the filled position when a wage agreement is reached. The larger the difference, the easier is for each to concede more surplus. A substantial wage increases the value of being employed to individuals, while it reduces the value of filling the vacancy to firms, and vice-versa. Hence if a public policy reduces their difference, they will try to compensate it by bargaining higher or lower wages respectively. Then each difference is then weighted by adding their respective bargaining parameter as an exponent. Finally, the product of the weighted differences represents the mutual gains from the agreement.

The wage agreed is a function not only of the bargaining parameter, but also of the variables and other parameters that determine the individuals' value of being employed or unemployed, and firms' value of filling the vacancy or not. The wage equation, along with the job creation equation determine the equilibrium wages and market tightness (figure 1.2). For example, an increase in unemployment insurance also increases the value of being unemployed, reducing the individual gains from employment, pushing wages up to compensate for that loss. Also, an increase in the bargaining power of individuals would increase the slope of the wage curve, increasing wages. All those changes also reduce the number of vacancies available to unemployed individuals, reflected in a market tightness decrease. This reduction in market tightness leads to higher unemployment. The effect of an increase in the wage curve slope is represented by a movement from point A to B in figure 1.2.

Figure 1.2 : Equilibrium wage and market tightness.



However, some of the parameters would change both curves given that the wages bargained depend on the profitability of the vacancy, and the profit maximization of the firm. For example, an increase in the costs of keeping the vacancy open increases the gains of filling it. Hence firms can offer larger wages for any given market tightness they face. Thus it increases the slope of the wage equation. At the same time, this also reduces the marginal profitability of the firm since it increases their costs. This shifts the job creation curve leftwards pushing wages down. The net result of wages is ambiguous, but vacancies and market tightness reduce undoubtedly since both increase the firms' costs. This equilibrium in represents by point C in figure 1.2.

1.3 Consumption-based Tax Reforms

1.3.1 Characteristics of Consumption-based Tax Reforms

A consumption-based tax is a tax which base is equivalent to a consumption tax one. The difference between income and consumption bases is their treatment of capital income. While they are taxed twice in the income tax –once when the income is received and again when capital income yield is withdrawn– a consumption tax is taxed only once (Gale 2005). Under the premise that any income is either consumed, saved or invested, a tax on income that is neither saved or invested is equivalent to a tax on today's consumption; and a tax on savings withdrawal or investment withdrawal, taxes future consumption. This approach is known as *consumed income* taxation. Another approach taxes all income once, avoiding taxing savings and investment yield again, known as *yield exemption.* In any case, capital income is taxed only once.[5]

[5]Consumption-based taxes exempt only normal (risk-free) returns; above-normal returns are taxed (Hubbard 2005, 83).

For the last three decades, fundamental consumption-based tax reforms have been proposed as an alternative to the current system based on the grounds of simplification, increased efficiency, and fairness (Auerbach and Hassett 2005; Henry J. Aaron and Steuerle 2007; Diamond and Zodrow 2008). The different treatment of savings by a consumption tax compared to an income tax system improves efficiency since it avoids any interference with individuals' allocation of consumption across time.[6] In consumption-based tax reforms, investments are fully expensed. Thus the effective marginal tax rate on them is zero. As a result, consumption-based taxes not only eliminate allocation inefficiencies among individuals' with tendencies to either save or consume but also enhance economic growth through increased investment rates.

However, only the investments produced after the reform enactment are expensed, there is an implicit tax on the capital invested before the adoption of the reform. Since new investments have higher returns given the tax expense, they comparatively reduce the return of previous investments, which continue paying taxes based on its depreciated value. Transitional rules –understood as extending depreciation rules for capital, used before the reform (Altig et al. 2001)– can be applied to prevent a one-time windfall to previously purchased capital. Progressivity of consumption taxes also has been a concern since the proportion of income destined to consumption is larger than the one to savings among lower income groups. Another critical component of the capital levy is its relationship with older generations which commonly own a larger share of invested capital.

Housing is considered an investment under consumption-based taxation. Therefore, housing can be taxed preemptively either when is bought or it can be expensed and the capital gains and imputed rent included in the tax base. In practice, however, all flat tax

[6]The efficient intertemporal consumption allocation assumes no change in interest rates and tax rates, and that individuals consume all their income in their lifetime (Brown 1948).

and X tax proposals excluded the imputed rent as a part of the base, given that it is not feasible to measure it accurately. Bradford's (2005) proposed solution was to tax housing and consumer durables as consumption goods. This is equivalent to deduct it as an investment, and then tax it at the moment of consumption as imputed rent. When the imputed rent is excluded from the tax base, there is a reduction in the tax burden for house owners compared with rental housing. Approximately 50 percent of the capital stock is composed by housing (Altig et al. 2001, n. 586), thus an exemption creates an extra levy on non-housing capital and a larger tax rate for everyone. On the other hand, the option to tax housing and is not popular among people since it increases prices in the housing sector.

1.3.2 Two-tier Consumption-based Taxes: The Flat Tax and the X-Tax

Two alternatives to proportional consumption taxes, the yield exemption taxes, are the flat tax and X tax. In the flat tax and the X-tax, labor expenses are deducted from the base and taxed directly to individuals, such as wages, salaries, and pensions. After the deduction, the base left –the business component– is taxed in a similar fashion to a proportional consumption tax; while the wages, salaries, and pensions –the compensation component (Bradford 2005)– are taxed at the individual level resembling an income tax. This separation provides an alternative that allows the introduction of progressivity in a more effective and efficient way than through rebates, differential commodity taxation, or only transfers. Both taxes exclude financial assets but include business ones.

The flat tax, proposed by Robert Hall and Alvin Rabushka (1983, 2007) applies the same proportional statutory rate to businesses and individuals. The business tax base is composed of the difference between all sales of goods and services, and purchases from other firms. Investments such as plant, equipment, land, intangibles such as copyrights and

Table 1.1 : Flat tax proposals.

Standard deduction	Hall & Rabushka	Rep. Armey	Sen. Shelby
Married individuals filling jointly	$14,501	$31,774	$32,386
Singles	$8,888	$15,887	$16,193
Heads of household	$13,098	$20,786	$20,675
Additional for each dependent	$1,754	$7,424	$6,976
Flat tax	19 %	17%[a]	17%[b]

[a] 20% for taxable years previous to Jan. 1998.
[b] 19% for taxable years previous to Jan. 2005.

patents, and housing rental are treated as expenses, but includes its profits from investments in plant, equipment, and land, and rental from housing or services.

Wages, salaries, and pensions deducted from the business component are taxed individually, including a standard deduction for the lowest income bracket to keep the tax progressive; that is, the effective tax rate for the lowest income bracket is zero. The compensation tax base includes the actual payment of wages, salaries, and pension receipts; savings are taxed at the receipt of the compensation, but not when withdrawn, hence they are not double taxed. Fringe benefits are included, while interest payments (and receipts), inheritances, and gifts are excluded. Flat tax proposals were introduced in legislation by Rep. Richard Armey (U.S. Congress 1995) and Sen. Richard Shelby (U.S. Congress 2003) in the U.S. Congress. Table 1.1 shows the flat tax proposals along with their corresponding standard deductions, in 2013 dollars.[7]

The X tax is a variation of the flat tax in the sense that has progressive marginal rates instead of a flat tax; proposed by David Bradford (1986, 329).[8] The compensation tax has a progressive marginal rate with a cap, for the highest income bracket, equal to the business component tax rate. It may include tax credits for low-income households such as the Earned Income Tax Credit (Bradford 2005, 14). The reason why the business component

[7] Calculated using the CPI Inflation Calculator of the Bureau of Labor Statistics. The original amounts can be found in Hall and Rabushka (1983, 35) and U.S. Congress (1995, 2003).

[8] See also Bradford (2005) and Carroll and Viard (2012) for an extended description of the X tax.

Table 1.2 : X-tax proposal.

Single filling income brackets	X tax
$<$ \$47,713	15%
\$47,713-\$68,587	25%
$>$ \$68,587	35%
Businesses	35 %

matches the cap of the compensation tier is that if the compensation has a larger marginal rate, then individuals can register as businesses and evade the difference. The X tax was studied as an alternative by the President's Advisory Panel on Federal Tax Reform (2005) under the name of Progressive Consumption Tax. Table 1.2 shows the progressive tax rates for single filling, in 2013 dollars; for married filing the income amounts double.[9]

Although it is known that flat rate consumption-based taxes are commonly regressive, some alternatives had introduced different degrees of progressivity at an individual level of taxation. In particular, the flat tax and the X tax are options that address the regressive nature of a consumption-based tax reform, while keeping its benefits on growth and efficiency at the same time.

1.4 A New Model To Analyze Efficiency and Incidence of Consumption Tax Reforms

1.4.1 The Unemployment Equilibrium Effects

Most of the literature studies of the effects of tax reforms on unemployment equilibrium had focused on labor tax changes (Pissarides 1998; Bovenberg, Graafland, and De Mooij 2000; Heijdra and Ligthart 2004; Michaelis and Birk 2006) and their progressivity (Pissarides

[9]Calculated using the CPI Inflation Calculator of the Bureau of Labor Statistics. The original amounts can be found in the President's Advisory Panel on Federal Tax Reform (2005, 183).

1998; Sørensen 1999). In a search-and-matching framework, the basic premise is that any labor substitution effect consequence of a tax change will also be accompanied by an effect on the unemployment rate in the same direction. That is, for example, if the tax increases and labor supply reduces, the level of unemployment reduces as well. The explanation is that a reduction in the after-tax wage reduces the cost of the individual to concede a lower pre-tax wage during the bargaining, while it increases the cost of the firm to increase the after-tax wages (Sørensen 1999, 445). In other words, for any percentage increase in wages, the percentage received by the individual is lower; thus, the increase will not change individual's utility as much. While for firms, for any percentage increase in the after-tax wages, the percentage increase in the pre-tax wage has to be larger; thus, it would reduce its marginal profit from a filled vacancy more than before. This translates to a reduction of the wage curve slope, which reduces wages and increases market tightness. The increase of market tightness increases the probability of being hired and reduces unemployment.

Pissarides (1998) shows that the magnitude of the effects of a change in wage taxes on wages and unemployment –a tax cut in that case– depends on the structure of the unemployment compensation. When the compensation is fixed the slope of the individual labor supply curve is less inclined than the case with a variable compensation. The policy consequences are that a tax cut with a variable unemployment compensation would have a larger effect increasing wages than reducing unemployment, transferring the burden of the tax cuts on the unemployed that have relatively a deterioration in their living standards. Similarly, he shows that a change in the structure of taxation, more or less progressive, does not have an effect on unemployment when the model is a competitive equilibrium one. In that case, the analysis of tax policy using different progressive systems would now show their difference in terms on how much unemployment can be reduced, vis-a-vis a reduction in work incentives and productivity, using the same structure of unemployment compensation. Sørensen (1999) finds that the degree of progressivity that can be achieved could be quite large, but that it depends on the level of unemployment compensation. The reasoning behind is that since in a model of imperfect labor competition and increased progressivity produces a trade-off between unemployment reduction along with less individual labor supply (effort and productivity). The consequences regarding policy are that an increased progressive system might have to be accompanied by a reduction in unemployment compensation to set off the effects or not increase unemployment. Picard and Toulemonde (2003) show that if taxes are progressive and nonlinear, the effects on unemployment are different; since the revenue effect might offset the wage effect and the increase in the average rate.

The effects of consumption-based tax reforms had not been tested using search-and-matching unemployment equilibrium with income-heterogeneous individuals. The wage effect is expected to be similar, however since the structure of unemployment insurance in U.S. is indexed by wages, it is anticipated to have a larger effect on wages than in unemployment (Pissarides 1998). Also, since the taxation is nonlinear, the revenue effect is expected to be larger than the wage effect (Picard and Toulemonde 2003). Also, the reduction of double taxation of capital income would reduce the tax base and increase unemployment (Michaelis and Birk 2006). However, since consumption tax reforms are usually accompanied by the elimination of deductions, depreciation allowances, and other base-broadening policies, any reduction in the tax base must be balanced with those base-broadening effects. On the individuals' side, the net effect depends on variables such as the income elasticity of labor supply. A correction on the level of this variable which empirically shows small would reduce the change this effect. But similarly, it reduces the impact that an increase in wages has on the incentives to work due to an increase in labor demand. This portion is corrected using a functional form for the utility that does not have an income elasticity of labor supply equal to one, more on the flavor of Sørensen (1999).

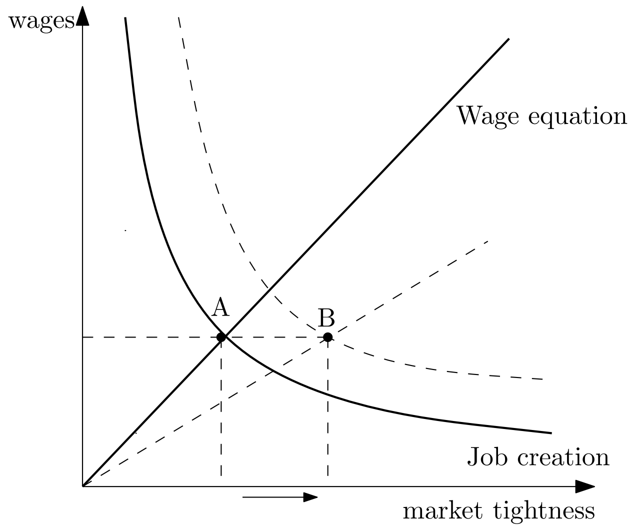
In the case that the consumption-based tax reform disincentives the individual labor supply, even by a small amount, it reduces the mark-up gains obtained from a filled vacancy, increasing the push to reduce the wages by firms. This moves slope of wage equation down, reducing wages and also increases market tightness since it would be profitable to increase the number of vacancies given the cost reduction. Also, enhances the effect produced by the reduction in the marginal gains of being employed by individuals, hence reducing unemployment more. However, the decrease in the slope is partially moderated by also a decrease in the marginal gains of a vacancy filled which gives the incentive to give up part of the profit for a higher wage.

Looking at the job creation equation, on one side this change in labor supply reduces its slope, hence reducing wages more and also market tightness since some vacancies will be closed due to some loss of the gains from keeping them filled. The net result would depend on which effect is larger, however, we have not considered yet the effects of other change of variables on the job creation equation such as investment expensing and its effect on capital accumulation. What seems to happen almost with no doubt is the reduction in wages. This added to the shift of the tax to the consumer can affect lower income brackets.

On the side of incidence, the effect of progressive taxation plays a major role. Progressive taxation increases also their incentive to work more than it does on higher income brackets. Similarly, the increase in the tax rate to have progressive taxation expands the effects mentioned in the previous paragraph related to the efficiency effects.

The previous analysis of the wage equation also considers the effect of a standard deduction or a progressive marginal income tax rate. For example, an increase in the marginal income tax progression reduces the marginal gains from bargaining and wages and in-

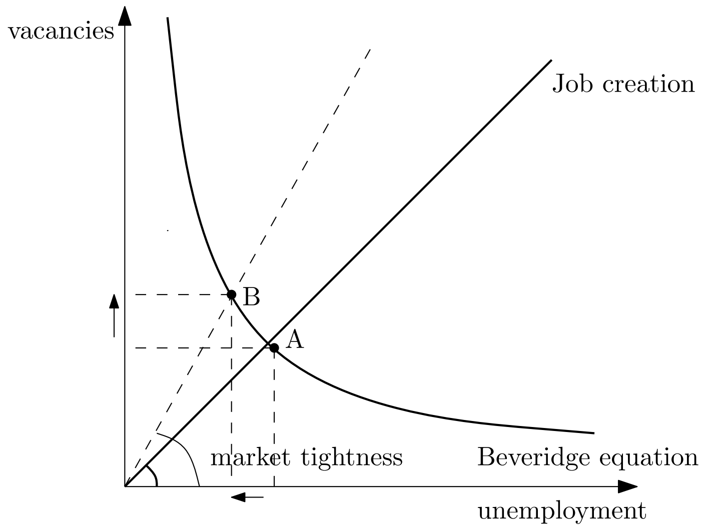
Figure 1.3 : Equilibrium wage and market tightness: effect of a increase in progressivity.



creases unemployment, but at the same time, it also alleviates firms from labor costs, potentially increasing the number of vacancies and the probability to get hired. Figure (1.3) shows the direct effect on the wage curve, and the second order effect on the individual labor supply, keeping the wage level constant. The X-tax and the flat tax have not been scrutinized using imperfect labor markets. The main difference with the results extracted using a competitive labor market come from the wage equation. Any effect in the marginal gains (obtained from employment or vacancy filling), that is, the rents bargained between individuals and firms, determine a wage response counter arresting this effect.

Similarly, a consumption-based reform is expected to increase capital and savings, through the expensing of firm's investment and exemption of savings taxation. On individuals, it increases the benefits of employment, increasing hours of work, and balancing the disincentives to individual labor supply. Hence, it produces an opposite effect that increases labor market tightness. On firms, it also reduces the marginal cost of capital, diminishing the previous effect. Depending which effect prevails over the market tightness, the consumption-based tax reform could increase or decrease the unemployment level. Rel-

Figure 1.4 : Equilibrium vacancies and unemployment: effect of a increase in market tightness.



evant parameters to determine the magnitude of the effects are the elasticity of labor supply, the elasticity of factor substitution, and the replacement rate. Figure (1.4) shows the case when the effect on employment is positive.

It also must be taken into account that now vacancies are considered by firms as investment projects that increase their effective labor, competing with capital for resources, and reducing the level of capital accumulation compared with a standard CGE model with a competitive labor market. Also, since the model only analyzes the state-state equilibrium, it misses the gains from reforms on old capital, besides the new investment expensing.

1.5 Conclusion

For all these reasons, a departure from the assumption of a competitive labor market is essential to CGE consumption-based tax reform modeling. The inclusion of structural unemployment into CGE modeling enriches the analysis of public policy's effects on unemployment and welfare, particularly the combined effect of tax and unemployment benefit policies. Given the different implications of public policy through imperfect labor market compared with a standard one, such inclusion provides a more accurate measurement of the effects of these policies on efficiency and incidence, and additional light about the advantages and disadvantages of different reforms (e.g. tax reforms) and its overall effect on welfare. Given the main role of tax policy and unemployment insurance in the search-and-matching framework, the proposed model can be used for both, tax and unemployment insurance, policy reform analysis. The objective of this chapter is to point out the advantages to use a model of this nature, mixing this two branches of literature. The model should be able to test if growth, capital accumulation and economic efficiency that typically occur under standard CGE analyses of consumption tax reforms are enhanced or reduced with the addition of a search-and-matching labor market framework. Also, analyze if such reforms are distributionally neutral or if they affect income groups disproportionately. Finally, by evaluating two consumption tax reforms with different progressive tax structures –which include only a standard deduction (the flat tax) or progressive marginal tax rates (the X-tax)– analyze the impact of such differences in the results.

Chapter 2

Evaluating Consumption Tax Reforms in a CGE Model with Unemployment and Income-Heterogeneous Individuals in the U.S.

2.1 Introduction

I simulate the effects of the enactment of two comprehensive consumption-based (or "consumption tax") tax reforms in the United States, a Hall-Rabushka style flat tax and the Bradford X-tax. Numerous studies have simulated the economic effects of such reforms in the U.S., but always within the context of a full employment model and often under the assumption of a single representative individual (or a single individual in each generation in the case of overlapping generations models). By comparison, my analysis includes a search-and-matching model of unemployment in the labor market with endogenous wage bargaining. Within this context, I examine the long-run effects of these reforms, including those in the labor market, as well as the distributional effects on twelve different income groups.

In a search-and-matching framework, labor market frictions result in unemployment in equilibrium, and the nature of that equilibrium is affected by the tax system. Consequently, the enactment of a tax reform such as the replacement of the income tax with a consumption tax could –in addition to its many other effects– increase economic efficiency by reducing unemployment. In this context, for example, if the reform leads to a reduction of the marginal utility that individuals obtain from an after-tax wage increase, it becomes optimal for firms and individuals to agree to a lower wage rate during the bargaining process, and potentially, for individuals to reduce their labor supply. At the same time, lower wages also could reduce the firm's marginal cost of hiring another individual, which leads to the opening of more vacancies, and a less congested market for individuals. As a result, the individuals' chance to be matched increases, and the unemployment rate declines. This effect on unemployment could mitigate the efficiency loss caused by any reduction of individuals' labor supply.

In the model I develop, the effects of the enactment of the various consumption tax reforms are disaggregated by income group, taking into account the fact that some groups will have larger labor supply responses than others, and experience different changes in after-tax wages. Therefore, the effects on each group's consumption and savings decisions will also be different. The estimated aggregate efficiency gains from reform (including those from unemployment reduction) and the incidence of the reform differ from those simulated in a model –with or without unemployment equilibrium– that assumes an exogenous wage profile that is estimated separately or assumes an income-homogeneous population. Additionally, average progressivity under the consumption tax reform will have a different impact on the bargained wage depending on whether it arises due only to a standard deduction (the flat tax) or due to progressive marginal tax rates (the X-tax).

The model is related to two well-established branches of the tax policy literature. The first analyzes the efficiency of consumption tax reforms and incidence in models with income-heterogeneous individuals but assumes competitive labor markets and full employment, where changes in labor supply are due primarily to individual optimization decisions regarding the choice between leisure and work/consumption. These studies miss any efficiency gains or losses due to changes in the level of unemployment and changes in the framework for wage bargaining. The second branch analyzes tax reforms using a search-and-matching unemployment equilibrium in the labor market and has been devoted almost exclusively to analyzing labor tax reforms, ignoring issues of income distribution and the general equilibrium effects of such reforms. The inclusion of a search-and-matching unemployment equilibrium and income-heterogeneous individuals in a CGE model thus provides a more robust framework for measuring the effects of tax reforms than those currently found in the literature. The analysis also provides policy makers with a better understanding of the consequences of consumption tax reforms, including the effects of changes in tax progressivity.

I first compare the effects of replacing the current progressive income tax system with a distribution-neutral and revenue-neutral progressive X-tax reform. I measure the neutrality in distribution by the equivalent variation of the tax reform on each income group. Under this scenario, the effects reflect the change to a consumption-tax base in a model with equilibrium unemployment, holding the degree of progressivity roughly constant. I then analyze the enactment of a flat tax, focusing on the effects of changes in progressivity, relative to the distributionally neutral X-tax reform. These simulations examine whether the increases in growth, capital accumulation and economic efficiency that typically occur under standard CGE analyses of such reform are enhanced or reduced remains in a search-and-marching labor market framework. It also examines whether these reforms can be distributionally neutral or if various income groups are being affected disproportionately.

The following section describes the search-and-matching framework utilized to model the current tax system and the consumption-based reforms analyzed, as well as the other features of the general equilibrium model of the economy that I use in the analysis. I then present and interpret the results of the simulations.

2.2 Model Structure

2.2.1 Individual's Utility and Budget

I assume a closed economy with a population of N individuals (who may be either employed or unemployed), and a single production sector that combines capital and labor to produce a single composite consumption good.[1]

Income groups

Individuals are distributed across twelve income brackets, indexed by $y = \{1, \ldots, 12\}$.[2] Each income group accounts for a decile of the population, except for income groups 1, 2, 11 and 12. The first and last income groups represent the lowest and highest two percent of earners, while the second and eleventh percentiles represent the other eight percent of the population in the first and last deciles. This approach allows me to analyze economic incidence at both extremes of the income distribution and thus better observe any reform-induced changes in income inequality.[3] The population of each income group is denoted by N_y, such that $\sum_y N_y = N$.

Utility function

I define individuals' utility over two periods. Current consumption, C_y^P, occurs in the first period which represents one year, and future consumption, C_y^F, occurs in the second period, which extends indefinitely from the end of the first period. Following Ballard et al. (1985), in the first period, individuals decide to allocate their income between current consumption

[1] The modeling of individuals and firms' behavior is an adaptation of the model constructed by Zodrow and Diamond (2013).

[2] See Pechman and Okner (2000) and Fullerton and Rogers (1993) for a discussion of the advantages of including annual and lifetime (respectively) income heterogeneity in incidence analysis.

[3] See, for example, Altig et al. (2001) and Zodrow and Diamond (2013).

and savings used to finance future consumption. In the initial equilibrium each individual finances an increment in the expected annual flow of future consumption (C_y^F) with the return of capital services of newly produced capital goods bought with current period savings. This approach assumes myopic expectations, under which the individual assumes all prices will remain constant; current decisions regarding consumption and savings are a function only of current prices. Savings in the current period are immediately transformed into real assets. The rental of those assets increases the expected annual flow of capital services for the rest of the second period, and that finances a constant increment in all future annual consumption, C_y^F.

The consumption function C_y reflects a constant elasticity of substitution between present (C_y^P) and future consumption (C_y^F), such that

$$C_y = \left[[\psi^C]^{\frac{1}{\xi}} [C_y^P]^{\frac{\xi-1}{\xi}} + [1 - \psi^C]^{\frac{1}{\xi}} [C_y^F]^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} . \tag{2.1}$$

The parameter $\psi^C \in [0, 1]$ represents individuals' weights on preferences for present and future consumption. The parameter $\xi \geq 0$ is the elasticity of substitution between present and future consumption with respect to the interest rate.

Individual preferences are represented by a Greenwood–Hercowitz–Huffman utility function denoted by U_y, which is increasing with respect to consumption utility, C_y, and decreasing with respect of hours of work, H_y, such that:

$$U_y = \frac{C_y^{1+\frac{1}{\eta}}}{1 + \frac{1}{\eta}} - \psi^H \frac{H_y^{1+\frac{1}{\gamma}}}{1 + \frac{1}{\gamma}}. \tag{2.2}$$

The parameter $\psi^H \in [0, 1]$ represents individuals' weight on labor. The parameter $\gamma \geq 0$ accounts for the intensity of the responsiveness of labor supply (at the intensive margin) to changes in after-tax wages. Larger values of γ imply larger values of the uncompensated (and compensated) wage elasticity, $\Theta_{H,w}$ (equation 2.50) and income elasticity of labor supply, $\Theta_{H,\Omega}$ (equation 2.51)Analogously, $\eta > 0$ reflects the intensity of responsiveness of consumption to good's prices (Keane 2011, p. 966).

The primary advantage of the utility function (equation 2.2), relative to the more standard constant elasticity of substitution utility function, is that it allows the income elasticity of labor supply to be different than one (equation 2.51), and closer to the values found in the empirical literature, between -0.1 and zero (McClelland and Mok 2012). This income elasticity of labor supply with respect to income, as well as the wage elasticities of labor supply and consumption, can differ across individuals at different income levels.

If an individual is unemployed, $H_y = 0$, only the total consumption component remains in the utility function:

$$U_y = \frac{C_y^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}. \tag{2.3}$$

Income and expenditures

Expenditures Consumers' present and future net gross expenditures on goods and services are equal to

$$[1+\tau^C]pC_y^P; \text{ and} \tag{2.4}$$

$$[1+\tau^C]pC_y^F, \tag{2.5}$$

where p denotes the price level of goods and services, and $\tau^C = \alpha^C \tau^{\text{con}}$ represents the fraction α^C of the tax τ^{con} charged to consumers. This formulation allows tax exemptions for certain goods - like medicine or food (e.g., as under current state sales taxes) - or for income or consumption tax preferences for the consumption of certain commodities, such as owner-occupied housing under the current income tax. The flat tax and X-tax replace the current tax charged to consumers, such that $\tau^{\text{con}} = 0$.

Wage income Wage income is equal to $w_y H_y$, where $w_y > 0$ is the wage hourly rate. The wage income tax base for an employed individual is equal to

$$B_y^e = w_y H_y - \omega^{\text{ded}}, \tag{2.6}$$

that is, gross wage income less the standard deduction. The current system, the X tax, and the flat tax all include a standard deduction, denoted by ω^{ded}.

Payroll taxes, τ_y^{pay} , include Social Security and Medicare taxes. I do not model on Social Security taxes so that all income groups are taxed proportionally.[4] Following Auerbach and Kotlikoff (1987), the marginal wage income tax is composed by a proportional rate, τ_y^{prp}, and a progressive factor, $\alpha^w B_y^e$ ($\alpha^w \geq 0$), such that

$$\tau_y^w = \tau_y^{\text{prp}} + \alpha_y^w B_y^e, \tag{2.7}$$

and $\alpha_y^w \leq \frac{1-\tau_y^{\text{prp}}}{B_y^e}$. In the case of the flat tax, the tax is proportional, that is, $\alpha_y^w = 0$. Wage income after income and payroll taxes, denoted by W_y^τ, is equal to:

$$W_y^\tau = w_y H_y - \int \tau_y^w dB_y^e - \tau_y^{\text{pay}} B_y^e, \tag{2.8}$$

[4] I also assume Social Security and Medicare taxes share the same wage income base. The Social Security tax rate is 6.2% with a cap at \$ 118,500 for 2016, and is matched by the employer. There is an additional 0.9% to 1.25% Medicare tax when wages pass \$ 200,000; these are not matched by the employer. See https://www.irs.gov/taxtopics/tc751.html.

where

$$\int \tau_y^w dB_y^e = \tau_y^{\text{prp}} B_y^e + 0.5\alpha_y^w [B_y^e]^2 \tag{2.9}$$

denote the amount of income tax collected. The average wage income tax in each income bracket, $\bar{\tau}_y$, is equal to

$$\bar{\tau}_y = \frac{\int \tau_y^w dB_y^e}{B_y^e} = \tau_y^{\text{prp}} + 0.5\alpha_y^w B_y^e. \tag{2.10}$$

The proportional component, τ_y^{prp}, is indexed by income group to adjust the average tax rate to the correct one for that particular group.

Savings Savings, S_y, buy an additional stream of future capital income that augments the future consumption stream in the initial equilibrium, C_y^F (Ballard et al. 1985). The price of savings is denoted by p^S. Thus, $p^S S_y$ is the value of savings, which equals the value of real asset investment. The flow of capital services produced with each unit of savings invested is denoted by κ (considered exogenous), which earns p^κ per unit. Consequently, the return of capital services from each unit of savings κ is $p^\kappa \kappa S_y$. This flow of capital income is spent on future consumption which costs pC_y^F. That is,

$$p^\kappa \kappa S_y = pC_y^F. \tag{2.11}$$

Equation 2.11 implies that $p^S S_y = \frac{p^S}{p^\kappa \kappa} pC_y^F$. In other words, the value of savings equals the present value of the increment in future consumption expenditures, where $r^S = \frac{p^\kappa \kappa}{p^S}$

denotes the real rate of return of those savings before taxes. Hence,

$$r^S p^S S_y = pC_y^F. \tag{2.12}$$

The capital income derived from savings is taxed at a rate τ^r under the current income tax system. Thus, the effective rate of return to savings is equal to $[1 - \alpha^r \tau^r]r^S$, where α^r indicates whether the tax is applied or not. Under any of the reform proposals, capital income is not included in the individual tax base, so $\alpha^r = 0$. Therefore, the future expenditure is financed with the after-tax return to investment,

$$[1 - \alpha^r \tau^r] r^S p^S S_y = [1 + \tau^C] pC_y^F. \tag{2.13}$$

Thus, the present value of the investment in future consumption is such that

$$p^S S_y = \frac{[1 + \tau^C] pC_y^F}{[1 - \alpha^r \tau^r] r^S}, \tag{2.14}$$

Non-wage income

Unemployment insurance When individuals are unemployed, the government provides unemployment compensation which equals a fraction of their wage income, $\rho_y w_y H_y$, with $\rho_y \in [0, 1]$ known as the *replacement rate*. This is the case of a replacement rate that indexes unemployment benefits to wages.[5] The replacement rate has a cap on the amount of benefits delivered, however, to simplify the simulation process, I assume a replacement rate that reflects the average ratio of unemployment benefits to wage income for each income group. Under the current system, the income tax base for unemployment equilibrium

[5] See Pissarides (1998) for the case when unemployment benefits are fixed; that is, not indexed by wages.

individuals is

$$B_y^u = \rho_y w_y H_y - \omega^{\text{ded}}. \tag{2.15}$$

Thus the after-tax income from unemployment insurance is

$$P_y^\tau = \rho_y w_y H_y - \int \tau_y^w dB_y^u. \tag{2.16}$$

Payroll taxes are not assessed on unemployment benefits.

Other benefits and assets Individuals also receive transfers, M_y, such as Social Security benefits, a portion α^M of which is financed with payroll taxes, which are taxed at a rate τ^M. Neither of the reforms exempts individuals from this tax.

Capital gains and dividends individuals receive are taxed under the current income tax system but not under any of the consumption tax reforms analyzed; any above-normal returns are taxed only once under the business cash flow tax. Finally, individuals also have an accumulated value of assets denoted by A_y, that yields the after-tax interest income $[1 - \alpha^r \tau^r] r A_y$, where r denotes the interest rate.

Budget constraint

An employed individual's intertemporal budget in steady state is equal to

$$[1 + \tau^C] p C_y^P + p^S S_y = W_y^\tau + \Omega_y.$$

That is,

$$[1+\tau^C]pC_y^P + \frac{[1+\tau^C]pC_y^F}{[1-\alpha^r\tau^r]r^S} = W_y^\tau + \Omega_y;$$

where

$$W_y^\tau = w_y H_y - [\bar{\tau}_y + \tau_y^{\text{pay}}]B_y^e, \text{ and} \tag{2.17}$$

$$\Omega_y = [1+\tau^M]M_y + [1-\alpha^r\tau^r]rA_y, \tag{2.18}$$

denote the income after wage income and payroll taxes, and the after-tax income from transfers and assets. Analogously, when the individual is unemployed, in the intertemporal budget constraint, P_y^τ replaces W_y^τ.

2.2.2 Firms' Production, Earnings and Value

Production function

Let $L = \sum_y N_y H_y$ denote the total number of hours available in the economy, and let e denote the employment rate. Firms produce aggregate output Q using capital K and employed labor eL as inputs in a CES production function,

$$Q = \left[[\psi^Q]^{\frac{1}{\epsilon}} K^{1-\frac{1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}}[eL]^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}; \tag{2.19}$$

where $\psi^Q \in [0,1]$ is the factor weighting parameter in the production function, and $\epsilon > 0$ is the elasticity of substitution between employed labor and capital with respect to the before-tax factor price ratio.

Earnings

As shown in equation 2.20 below, firm earnings before depreciation and corporate income taxes, π, are equal to the revenue from sales less labor costs, interest payments on debt, and property taxes. Labor costs include wages paid to employees at an hourly wage $\bar{w} = \sum_y N_y w_y / N$, and employment taxes at rate τ^u, which include both unemployment insurance and Social Security contributions.[6] In addition, as described below, the model includes the cost associated with posting job vacancies cvN, where c represents the average cost to keep a vacancy open; and the number of vacancies vN is a fraction v of the population. For simplicity, I do not include business purchases of goods, services, and materials necessary to produce Q. The amount of debt, D, is assumed to be a fixed proportion, α^D, of the firm's capital stock, so that $D = \alpha^D K$. Interest expense on this debt is rD. Firms also pay property taxes, which tax rate is denoted by τ^{pro}. Thus, earnings before depreciation and taxes are:

$$\pi = pQ - [1 + \tau^u]\bar{w}eL - cvN - \tau^{\text{pro}}K - rD. \tag{2.20}$$

The firm also invests amount I. The capital stock value increases with investment, and declines over time due to depreciation, which occurs at rate $\delta \in [0, 1]$, such that

$$\Delta K = I - \delta K. \tag{2.21}$$

[6]This includes the employer portion of social security taxes. Unemployment insurance for firms has a salary cap, which I assume that is captured by the level of τ^u. I also assume τ^u includes the federal taxes and an average of state unemployment insurance taxes.

In steady state $I = \delta K$ (there is no growth in the model), and the firm invests only the fraction of its capital lost by depreciation. Because ΔK is zero in steady state and the debt to capital ratio is constant, the issuance of new bonds in steady state is also zero.

The business income tax base, B^{π} is defined as

$$B^{\pi} = pQ - \tau^{\text{pro}}K - [1 + \tau^{u}]\bar{w}eL - cvN - \alpha^{k}\delta^{\tau}K - \alpha^{I}I - \alpha^{d}rD; \tag{2.22}$$

where δ^{τ} is depreciation for tax purposes and $\alpha^{k}, \alpha^{I}, \alpha^{d} \in [0, 1]$ indicate whether an item is included in the tax base. The current corporate income tax system taxes business profit but includes some tax preferences or business tax expenditures, includes depreciation allowances and interest expense deductions, and does not allow expensing of investment; hence, $\alpha^{I} = 0$, $\alpha^{d} = 1$, and $\alpha^{k} = 1$. The flat tax and the X tax include expensing of investment, and eliminate depreciation allowances and interest deductions, hence, $\alpha^{I} = 1$, $\alpha^{d} = 0$, and $\alpha^{k} = 0$. The income tax, flat tax and the X tax all allow deductions for labor expenses, with labor income expenses taxed at the individual level. Denoting the business tax rate as τ^{π}, after-tax earnings net of economic depreciation are

$$\pi - \tau^{\pi}B^{\pi} - \delta K. \tag{2.23}$$

Dividends, X, are assumed to be a constant fraction, $\alpha^{X} \in [0, 1]$ of those earnings.

Firm's value

New share issues are used to finance any investment that cannot be financed with debt or with the firm's net cash flow, that is, the difference between the after-tax earnings net of dividend payments, such that

$$\pi - \tau^{\pi} B^{\pi} - X + Z = I. \tag{2.24}$$

That is,

$$Z = I - [1 - \alpha^{X}][\pi - \tau^{\pi} B^{\pi}] - \alpha^{X} \delta K. \tag{2.25}$$

In other words, if cash flow is insufficient to pay dividends, the firm finances the gap by selling new equity shares, collecting the amount $Z > 0$; if cash flow is large enough to pay dividends, the firm buys shares, returning the extra cash to stock holders in the amount $Z < 0$.

Let Π denote the value of the firm, τ^X the dividend tax rate, and τ^G the annual accrual tax rate on capital gains, both taxed at the firm level; $\tau^G < \tau^X$ to reflect the benefits of tax deferral (due to taxation of capital gains only upon realization), and exemption when gains are transferred at death, which is only partially offset by the taxation of inflationary gains, especially in a relatively low inflation environment (Diamond and Zodrow 2005). None of the tax reforms provide for taxation of dividends or capital gains. The individual level no-arbitrage condition (Goulder and Summers 1989), requires

$$[1 - \alpha^{r} \tau^{r}] r = \frac{[1 - \alpha^{x} \tau^{X}] X + [1 - \alpha^{g} \tau^{G}][\Delta\Pi - Z]}{\Pi}; \tag{2.26}$$

using equation 2.26, the value of the firm in steady state is equal to:

$$\Pi = \alpha^{v} \left[\alpha^{\pi} [\pi - \tau^{\pi} B^{\pi} - \delta K] + \delta K - I \right] \tag{2.27}$$

where

$$\alpha^v = \frac{1-\alpha^g\tau^G}{[1-\alpha^\tau\tau^r]r} \tag{2.28}$$

$$\alpha^\pi = \frac{\left[\alpha^X[1-\alpha^x\tau^X]+[1-\alpha^X][1-\alpha^g\tau^G]\right]}{1-\alpha^g\tau^G}; \tag{2.29}$$

and α^x and α^g indicate whether dividends and capital gains are taxed, respectively. Under the current income tax system $\alpha^x = \alpha^g = 1$, and under both consumption tax reforms, $\alpha^x = \alpha^g = 0$.

2.2.3 Search-and-Matching Labor Market

Matching and Unemployment

The firms in the model announce vacancies to recruit candidates for job positions, to which unemployed individuals apply. The labor market matches only a fraction of vacancies and unemployed individuals due to labor market frictions.[7]The unemployment, job-vacancy, and employment rates with respect to N are denoted u, v, and $e = 1-u$, respectively. The quantity of individuals and vacancies matched, or employment inflow, is modeled using a Cobb-Douglas (Pissarides 2000) matching function with constant returns to scale

$$m = \mu u^\beta v^{1-\beta}, \tag{2.30}$$

where m is the fraction of the unemployed population that is matched to a job vacancy, $\mu > 0$ is the parameter that measures the efficiency of the market matching job applicants and openings, and $\beta \in [0,1]$ is the elasticity of matches with respect to the unemployment

[7]The labor market is modeled following Pissarides (2000).

rate.[8] The matching efficiency parameter captures factors such as the availability of virtual or physical spaces that facilitate labor exchange, or the compatibility between the skills offered by individuals and the ones demanded by the firms. The parameter β represents the sensitivity of the matching rate to the positive (and negative) externalities created by a relative increase in the number of firm vacancies or unemployed individuals. A larger number of unemployed individuals implies less competition among firms in filling their vacancies, but more competition among the unemployed. A large β enhances the positive effect on firms, while it diminishes the negative effect on unemployed individuals. Similarly, a larger number of vacancies implies less competition among unemployed individuals, but more competition among firms. In that case, a large value of $1 - \beta$ enhances the positive effect on unemployed individuals, and diminishes the negative effect on firms (Petronglo and Pissarides 2001).

At the same time, employment outflow is affected by exogenous shocks that occur at a fixed *separation rate*, λ; this assumption simplif ies the model, which focuses on the process of job creation.[9] As a result, the change in the number of employed people, ΔeN, is equal to the difference between the employment inflow and outflow,

$$\Delta eN = mN - \lambda eN; \tag{2.31}$$

in steady state, unemployment inflow and outflow are equal.

A common way to represent market congestion is market *tightness*, which is defined from the firm perspective as ratio of the vacancy rate to the unemployment rate, which re-

[8] A matching function is a concave function that is continuous, non-negative, and increasing in both arguments. It is usually assumed to be homogeneous of degree one, and such that $m(u, 0) = m(0, v) = 0$ (Pissarides 2000). The Cobb-Douglas is the most popular function, and the parameters estimated from data are consistent with the constant returns to scale functional form (Petronglo and Pissarides 2001).

[9] See Pissarides (2000) for endogenous separation rates as function of a drop in the job productivity below a threshold determined by the firm, or by individuals' on-the-job search.

flects the congestion, or tightness that firms face when the number of job vacancies relative to the number of unemployed workers increases. Market tightness, θ, is a function of the inputs of the matching function, or

$$\theta = vN/uN. \tag{2.32}$$

Firms are said to face a *tight* labor market when the number of job vacancies is larger than the number of unemployed workers, $\theta > 1$. The matching function, expressed as a function of market tightness, is

$$m = \mu\theta^{-\beta}v = \mu\theta^{1-\beta}u. \tag{2.33}$$

Firms finds a suitable worker with a probability equal to the number of job matches to the number of vacancies, $mN/vN = \mu\theta^{-\beta}$; similarly, workers are hired with probability $mN/uN = \mu\theta^{1-\beta}$. An increase in market tightness reduces the firm's probability of filling a vacancy. Conversely, an increase in the labor market tightness increases individuals' probability of being hired.[10]

The change in employment, ΔeN, expressed as a function of market tightness becomes, $\mu\theta^{-\beta} - \lambda eN$ (Pissarides 2000). In steady state, the employment rate is a function of the vacancy and separation rates, and labor market tightness:

$$e = \left[\frac{\mu\theta^{-\beta}}{\lambda}\right]v, \tag{2.34}$$

[10]In the simulations, section 2.4, I do not restrict the probabilities to be lower than one; however, that is not necessary since their respective simulated values are below that threshold.

which shows the role of vacancies in job creation and employment. Furthermore, vacancies can be expressed as a relationship between γ and unemployment

$$v = \left[\frac{[1-u]\lambda}{\mu u^{\beta}}\right]^{\frac{1}{1-\beta}} \tag{2.35}$$

Equation 2.35 shows that after firms choose their optimal vacancy rate they still need to face the job market to see how many of these vacancies will be left unfilled due to market frictions.

Another way to examine the equilibrium is through the change in unemployment,

$$\Delta u N = \lambda[1-u]N - \mu\theta^{1-\beta}uN. \tag{2.36}$$

In steady state, the unemployment inflow and outflow are the same, leading to an unemployment rate equal to

$$u = \frac{\lambda}{\lambda + \mu\theta^{1-\beta}}. \tag{2.37}$$

This is the *Beveridge* equation (Pissarides 2000), which shows the relationship between the unemployment rate, market tightness, and the employment separation rate.

Wage Bargaining

Within this context, firms and individuals optimize, with individuals choosing to be employed or unemployed and firms choosing to fill or leave unfilled their job vacancies. The Bellman equations determine the value of being in each state as the sum of the current value of being in that state and the discounted expected value in the next period. The expected value is calculated using the separation rate, λ, the probability of filling a vacancy, $\mu\theta^{-\beta}$, and probability of finding finding a job, $\mu\theta^{1-\beta}$. Assume a discount factor for workers and firms equal to $[1 + r^{\tau}]^{-1}$. The steady state Bellman equations, which show the value of being employed V_y^e or unemployed V_y^e (Boeters and Savard 2013), are:

$$V_y^e = U_y^e + [1 + r^{\tau}]^{-1}[\lambda V_y^u + [1 - \lambda]V_y^e], \text{ and} \tag{2.38}$$

$$V_y^u = U_y^u + [1 + r^{\tau}]^{-1}[\mu\theta^{1-\beta}V_y^e + [1 - \mu\theta^{1-\beta}]V_y^u] \tag{2.39}$$

Individuals keep working as long as $V_y^e \geq V_y^u$. Individual indirect utility functions are denoted by U_y^e and U_y^u, under employment and unemployment states respectively. Similarly, the values of a filled vacancy, V^f, and an unfilled vacancy, V^v, are:

$$V^f = \Pi^f + [1 + r^{\tau}]^{-1}[\lambda V^v + [1 - \lambda]V^f], \text{ and} \tag{2.40}$$

$$V^v = -c + [1 + r^{\tau}]^{-1}[\mu\theta^{-\beta}V^f + [1 - \mu\theta^{-\beta}]V^v] \tag{2.41}$$

where $\Pi^f = \partial\Pi/\partial eN$ denotes the marginal increase in the value of the firm due to an additional employee.

The wage determination process has its origin in bargaining theory (Nash 1950; Binmore, Rubinstein, and Wolinsky 1986; Shaked and Sutton 1984). The idea behind wage setting is that the labor exchange produces a surplus, and the wage set determines how the surplus will be split between employees and firms. Labor market frictions create monopolistic power during the job matching (Pissarides 2000), allowing firms and individuals to bargain for a larger share of surplus. The equilibrium wage, represented in the *wage equation*, maximizes the joint gains of firms and individuals extracted from their job agreement, weighted by the respective power bargaining power of each group, which corresponds to the *Nash bargaining solution*. In equilibrium the marginal benefits and losses, either to individuals and firms, from a change in wages are offset. The equilibrium wage rate has the form:

$$w = \arg\max_{w} [V_y^e - V^u]^\sigma [V^f - V^v]^{1-\sigma}, \tag{2.42}$$

where $\sigma \in [0, 1]$ represents the worker's bargaining power. Wages are denoted simply as w under the assumption that firms are concerned only about aggregate wages and do not distinguish the income group to which an individual belongs. Thus while an individual of income group y bargains for wage w_y, the firm bargains for $\bar{w}$, both here represented by w. The resulting set of bargaining wages is equal to $\{w_y\}_{\forall y}$, and $\bar{w} = \sum_y N_y w_y / N$.

Tax reforms affect the value of employment, which is reflected in wages, to different degrees depending on the income group to which an individual belongs. In addition, wages are also affected by unemployment insurance, which affects the cost of unemployment.

2.2.4 Government's Budget

Finally, the government is assumed to be subject to a balanced budget constraint, and each one of the reforms are revenue-neutral according to this budget. There are two different levels:

- The amount collected from payroll taxes and firm's contributions should cover expenses on transfers and unemployment benefits, such that:

$$\sum_y e N_y \tau^{\text{pay}} B_y^e + \tau^u \bar{w} e L = \sum_y N_y \alpha^M M_y + \sum_y u N_y \rho_y w_y H_y. \tag{2.43}$$

- The rest of the government expenses, pG, equals the rest of its revenue:

$$\begin{aligned} pG = &\sum_y N_y \left[\tau^C p \left[eC_y^{Pe} + uC_y^{Pu}\right] + \frac{\tau^C p}{[1-\alpha^r\tau^r]r^S}\left[eC_y^{Fe} + uC_y^{Fu}\right]\right] \\ &+ \sum_y N_y \alpha^r \tau^r r^S p^S \left[eS_y^e + uS_y^u\right] \\ &+ \sum_y N_y \left[\alpha^r \tau^r r A_y + \tau^M M_y\right] + \sum_y N_y \bar{\tau}_y \left[eB_y^e + uB_y^u\right] \\ &- \sum_y N_y [1-\alpha^M] M y \\ &+ \tau^\pi B^\pi + \tau^{\text{pro}} K + \alpha^x \tau^X X + \alpha^g \tau^G [-Z] \end{aligned} \tag{2.44}$$

2.3 General Equilibrium

The derivation of the general equilibrium equations is detailed in the Appendix A.

2.3.1 Individuals and Firms

Individuals

Employed Individuals maximize their utility given their budget constraints. Thus, individual choices regarding consumption, savings, and labor hours depend on employment status. I denote the optimal consumption, savings, and hours supplied when the individual is employed as C_y^{Pe}, C_y^{Fe} , S_y^e and H_y^e, respectively. The system of equations that solves the individual's maximization problem is

$$C_y^{Pe} = \alpha^\psi \frac{W_y^\tau(H_y^e) + \Omega_y}{[1+\tau^C]p} \tag{2.45}$$

$$C_y^{Fe} = [1-\alpha^\psi]\frac{[1-\alpha^r\tau^r]r^S[W_y^\tau(H_y^e)+\Omega_y]}{[1+\tau^C]p} \tag{2.46}$$

$$S_y^e = \frac{r^S}{p^\kappa \kappa}[1-\alpha^\psi][W_y^\tau(H_y^e)+\Omega_y] \tag{2.47}$$

$$H_y^e = \left[\frac{W_y^\tau(H_y^e)+\Omega_y}{[1+\tau^C]p}\right]^{\frac{\gamma}{\eta}} \left[\frac{\psi^C}{\alpha^\psi}\right]^{\frac{\gamma[1+\eta]}{\eta[\xi-1]}} \left[\frac{[1-\tau_y^w(H_y^e)-\tau_y^{\text{pay}}]w_y}{\psi_y^H[1+\tau^C]p}\right]^\gamma; \tag{2.48}$$

where

$$\alpha^\psi = \frac{\psi^C}{\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}}, \tag{2.49}$$

represents the share of income used in present consumption, expressed in terms of individual preference parameters, opportunity costs, and the proportion of tax on savings.

I also calculate the uncompensated wage and income elasticities of labor supply and the wage elasticity of consumption, used in the wage equation. The uncompensated wage elasticity of labor supply, Θ_{Hw}, is equal to

$$\Theta_{Hw} = \frac{\gamma\left[1-\frac{\alpha_y^w w_y H_y^e}{1-\tau_y^w-\tau_y^{\text{pay}}}\right] + \frac{\gamma}{\eta}\left[\frac{[1-\tau_y^w-\tau_y^{\text{pay}}]w_y H_y^e}{W_y^\tau+\Omega_y}\right]}{1+\gamma\left[\frac{\alpha_y^w w_y H_y^e}{1-\tau_y^w-\tau_y^{\text{pay}}}\right] - \frac{\gamma}{\eta}\left[\frac{[1-\tau_y^w-\tau_y^{\text{pay}}]w_y H_y^e}{W_y^\tau+\Omega_y}\right]}. \tag{2.50}$$

The income elasticity of labor supply, $\Theta_{H\Omega}$, is equal to

$$\Theta_{H\Omega} = \frac{\frac{\gamma}{\eta}\left[\frac{\Omega}{W_y^\tau+\Omega_y}\right]}{1+\gamma\left[\frac{\alpha^w w_y H_y^e}{1-\tau_y^w-\tau_y^{\text{pay}}}\right] - \frac{\gamma}{\eta}\left[\frac{[1-\tau_y^w-\tau_y^{\text{pay}}]w_y H_y^e}{W_y^\tau+\Omega_y}\right]}, \tag{2.51}$$

and the wage elasticity of consumption, Θ_{Cw}, is

$$\Theta_{Cw} = \frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y H_y^e}{W_y^\tau + \Omega_y}\left[1+\Theta_{Hw}\right]. \tag{2.52}$$

Unemployed Analogously, unemployed individuals' present consumption, C_y^{Pu}, future consumption, C_y^{Fu}, and savings, S_y^u, are

$$C_y^{Pu} = \alpha^\psi \frac{P_y^\tau + \Omega_y}{[1+\tau^C]p} \tag{2.53}$$

$$C_y^{Fu} = [1-\alpha^\psi]\frac{[1-\alpha^r\tau^r]r^S[P_y^\tau + \Omega_y]}{[1+\tau^C]p} \tag{2.54}$$

$$S_y^u = \frac{r^S}{p^\kappa\kappa}[1-\alpha^\psi][P_y^\tau + \Omega_y]. \tag{2.55}$$

Firms

Firms choose their optimal vacancy rate and stock of capital to maximize their value. The optimal vacancy rate and thus the level employment are determined in the labor market equilibrium. The optimal capital stock is

$$K = \frac{\psi^Q eL}{[1-\psi^Q]N}\left[\frac{\phi^K + \delta\phi^I}{\phi^L}\right]^{-\epsilon} \tag{2.56}$$

where

$$\phi^I = \frac{1}{1-\tau^\pi}\left[\frac{1}{\alpha^\pi} - \tau^\pi[\alpha^I + \alpha^k]\right], \tag{2.57}$$

$$\phi^K = \frac{1}{1-\tau^\pi}\left[[1-\alpha^d\tau^\pi]r\alpha^D + \delta - \frac{\delta}{\alpha^\pi}\right] + \tau^{\text{pro}}, \text{ and} \tag{2.58}$$

$$\phi^L = \frac{\lambda}{\mu\theta^{-\beta}}\frac{cN}{L} + [1+\tau^u]\bar{w}. \tag{2.59}$$

represent after-tax marginal costs of capital and labor respectively. Additionally, total output value yield

$$Q = \frac{eL}{1-\psi^Q}\left[\frac{1-\psi^Q}{1-\alpha^Q}\right]^{\frac{\epsilon}{\epsilon-1}} \tag{2.60}$$

where

$$\alpha^Q = \frac{\psi^Q[\phi^K+\delta\phi^I]^{1-\epsilon}}{\psi^Q[\phi^K+\delta\phi^I]^{1-\epsilon}+[1-\psi^Q][\phi^L]^{1-\epsilon}}. \tag{2.61}$$

Market clearing

The market clearing condition states that in each sector the sum of goods consumed must be the same as the ones produced, or

$$\sum_y N_y\left[e[C_y^{Pe}+C_y^{Fe}]+u[C_y^{Pu}+C_y^{Fu}]\right]+G+\sum_y N_y[1-\alpha^M]M_y = Q \tag{2.62}$$

Equation 2.62, along with the government budget determine the equilibrium values of prices and the interest rate, which depends on the labor market variables, wages and market tightness.

2.3.2 Job Creation and Wage Determination

The marginal increase in the value of the firm due to an increase of employed individuals is equal to:

$$\Pi^f = \alpha^v\alpha^\pi[1-\tau^\pi]\left[p\frac{\partial Q}{\partial eN}-[1+\tau^u]\bar{w}\frac{L}{N}\right], \tag{2.63}$$

where

$$\frac{\partial Q}{\partial eN} = \left[\frac{1-\alpha^Q}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}} \frac{L}{N} \tag{2.64}$$

Equation 2.63 shows the firm's mark-up obtained from a filled vacancy, that is, the difference between labor's marginal profit and wage-compensation costs, after-taxes. An increase in wages reduces the mark-up directly by increasing costs of workers' compensation, and indirectly by reducing its marginal profit. In a competitive labor market the mark-up equals zero; however, in a search-and-matching framework, the mark-up must be positive, and large enough to cover the costs of recruiting job candidates (as explained in the following section).

Free-entry Condition and Job Creation Equation

Solving the system of Bellman equations (equations 2.38, 2.39, 2.40, and 2.41) leads to

$$V_y^e = \frac{1+r^\tau}{r^\tau}\left[\frac{[r^\tau + \mu\theta^{1-\beta}]U_y^e + \lambda U_y^u}{r^\tau + \mu\theta^{1-\beta} + \lambda}\right], \tag{2.65}$$

$$V_y^u = \frac{1+r^\tau}{r^\tau}\left[\frac{\mu\theta^{1-\beta}U_y^e + [r^\tau + \lambda]U_y^u}{r^\tau + \mu\theta^{1-\beta} + \lambda}\right], \tag{2.66}$$

$$V^f = \frac{1+r^\tau}{r^\tau}\left[\frac{[r^\tau + \mu\theta^{-\beta}]\Pi^f - \lambda c}{r^\tau + \mu\theta^{-\beta} + \lambda}\right], \text{ and} \tag{2.67}$$

$$V^v = \frac{1+r^\tau}{r^\tau}\left[\frac{\mu\theta^{-\beta}\Pi^f - [r^\tau + \lambda]c}{r^\tau + \mu\theta^{-\beta} + \lambda}\right]. \tag{2.68}$$

Note that if an open vacancy can produce a positive return, $V^v > 0$, more firms will participate in the labor market until profit is reduced to zero; the case when $V^v < 0$ is analogous. Thus, in equilibrium $V^v = 0$, which is known as the *free entry* condition. This restriction leads to an equilibrium between the firm's mark-up obtained from a filled vacancy, Π^f, and the capitalized costs incurred when it fails to find a match, balancing the gains and losses from posting a vacancy. Such equilibrium is also known as the job creation equation,

$$\Pi^f = \frac{[r^\tau + \lambda]c}{\mu\theta^{-\beta}}. \tag{2.69}$$

After replacing Π^f with equation 2.63, it becomes

$$\alpha^\pi[1-\tau^\pi]\left[p\left[\frac{1-\alpha^Q(\theta)}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}} - [1+\tau^u]\bar{w}\right]\frac{L}{N} = \frac{[r^\tau + \lambda]c}{\mu\theta^{-\beta}}. \tag{2.70}$$

Vacancies are analogous to an investment project that yields positive returns or losses, and that the optimal vacancy rate maximizes its net expected return (Pissarides 2010). The free-entry equilibrium of benefits and costs of equation 2.70 shows how the costs of opening a vacancy increase with a larger market tightness, as firms take more time to find a good match, which increases recruiting costs. The job creation equation also shows how the marginal value of a filled vacancy declines when the marginal labor cost increases, for example, due to an increase in market tightness or wages, or when the marginal cost of capital decreases, which makes capital investment more attractive. The job and Beveridge equations determine the market tightness, and the unemployment and vacancy rates in terms of wages, prices and the interest rate.

Wage Determination

From the the first order condition of equation 2.42, we obtain the *wage equation*:

$$[1-\sigma]\left[U_y^e - U_y^u\right] = \sigma\left[\frac{\partial U_y^e/\partial w_y}{-\partial\Pi^f/\partial\bar{w}}\right]\left[\Pi^f + c\right], \tag{2.71}$$

which can also be expressed as the *surplus* needed in the wage agreement to change from an unemployed to an employed status

$$U_y^e = U_y^u + \frac{\sigma}{1-\sigma}\left[\frac{\partial U_y^e/\partial w_y}{-\partial \Pi^f/\partial \bar{w}}\right]\left[\Pi^f + c\right] \tag{2.72}$$

or, from the firm perspective, the one needed to fill the vacancy instead of leave it open,

$$\Pi^f = -c + \frac{1-\sigma}{\sigma}\left[\frac{-\partial \Pi^f/\partial \bar{w}}{\partial U_y^e/\partial w_y}\right]\left[U_y^e - U_y^u\right] \tag{2.73}$$

Equation 2.74, the marginal utility of employment with respect to wages, expressed in terms of consumption when the individual is employed and hours of labor, and wage elasticities of labor and consumption is

$$\frac{\partial U_y^e}{\partial w_y} = \frac{1}{w_y}\left[[C_y^e]^{1+\frac{1}{\eta}}\Theta_{C,w} - \psi^H [H_y^e]^{1+\frac{1}{\gamma}}\Theta_{H,w}\right], \tag{2.74}$$

and the marginal change in rents due to changes in wages, using the values of α^Q and ϕ^L is

$$\frac{\partial \Pi^f}{\partial \bar{w}} = \alpha^\pi[1-\tau^\pi][1+\tau^u]\frac{L}{N}\left[p\left[\frac{1-\alpha^Q}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}}\frac{\alpha^Q}{\phi^L} - 1\right]. \tag{2.75}$$

2.3.3 Equilibrium

In summary, the equilibrium system is comprised of the Beveridge equation (2.37) (BE), the individual labor supply equation (2.48) (LS), the market clearing condition (2.62) (MC), the government balanced budget (A.78) (GB), the job creation equation (2.70) (JC), and the wage equation (2.71) (WE). The general equilibrium system can be solved for the tuple

(w_y, H_y^e, r, θ); that is:

$$\text{BE}: \lambda[1-u] - \mu\theta^{1-\beta}u = 0 \tag{2.76}$$

$$\text{LS}: H_y^e - \left[\frac{W_y^\tau + \Omega_y}{[1+\tau^C]p}\right]^{\frac{\gamma}{\eta}} \left[\frac{\psi^C}{\alpha^\psi}\right]^{\frac{\gamma[1+\eta]}{\eta[\xi-1]}} \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y}{\psi_y^H[1+\tau^C]p}\right]^\gamma = 0; \tag{2.77}$$

$$\begin{aligned} \text{MC}: &\sum_y N_y \left[e[C_y^{Pe} + C_y^{Fe}] + u[C_y^{Pu} + C_y^{Fu}]\right] \\ &+ pG + \sum_y N_y[1-\alpha^M]M_y - Q = 0 \end{aligned} \tag{2.78}$$

$$\begin{aligned} \text{GB}: &\sum_y N_y \left[\tau^C p\left[eC_y^{Pe} + uC_y^{Pu}\right] + \frac{\tau^C p\left[eC_y^{Fe} + uC_y^{Fu}\right]}{[1-\alpha^r\tau^r]r^S}\right] \\ &+ \sum_y N_y \alpha^r \tau^r r^S p^S \left[eS_y^e + uS_y^u\right] \\ &+ \sum_y N_y \left[\alpha^r\tau^r r A_y + \tau^M M_y\right] + \sum_y N_y \bar{\tau}\left[eB_y^e + uB_y^u\right] \\ &- \sum_y N_y[1-\alpha^M]My \\ &+ \tau^\pi B^\pi + \tau^{\text{pro}}K + \alpha^x\tau^X X + \alpha^g\tau^G[-Z] - pG = 0 \end{aligned} \tag{2.79}$$

$$\text{JC}: \alpha^v\alpha^\pi[1-\tau^\pi]\frac{L}{N}\left[p\left[\frac{1-\alpha^Q}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}} - [1+\tau^u]\bar{w}\right] - \frac{[r^\tau + \lambda]c}{\mu\theta^{-\beta}} = 0 \tag{2.80}$$

$$\text{WE}: \sigma\frac{\partial U_y^e}{\partial w_y}\left[\Pi^f + c\right] + [1-\sigma]\frac{\partial \Pi^f}{\partial \bar{w}}\left[U_y^e - U_y^u\right] = 0 \tag{2.81}$$

BE and JC can both be expressed in terms of u and v and determine the equilibrium unemployment and vacancy rates, $u(w_y, p, r)$ and $v(w_y, p, r)$, as well as market tightness $\theta(w_y, p, r)$ and the employment rate, $e(w_y, p, r)$. Substituting into MC, GB and WE yields a system of three equations and three unknown variables $w_y(p, r)$, $p(w_y, r)$, and $r(w_y, p)$. Finally, the equilibrium is complete when these conditions are satisfied for each cohort y, which yields all the firm-related variables dependent of $\bar{w}$.

2.4 Simulations

2.4.1 Calibration

I use data from Statistics of Income by the Internal Revenue Service (IRS) (Internal Revenue Service 2013), and seasonally adjusted U.S. Bureau of Labor Statistics (BLS) data (U.S. Department of Labor 2015d; U.S. Department of Labor 2015c; U.S. Department of Labor 2015b; U.S. Department of Labor 2015a) for the estimation of income and consumption variables and the calibration of parameters. I assume a working population of $N = 145,238,285$, which is the total number of returns filled with the IRS in 2013. N is normalized to $\hat{N} = 1$. I also use Adjusted Gross Income (AGI) is used to divide the population into income groups and measure other variables, including wages and salaries, unemployment compensation, taxable income, interest income, marginal income tax, or Social Security benefits for each income group using a polynomial fit. Additionally, the level of Social Security benefits is adjusted to clear the goods market in the benchmark equilibrium. The benchmark equilibrium marginal income tax schedule is estimated using a weighted average of single, married and household marginal income tax schedules. I also choose $p^{\kappa}\kappa = r^{S} = r$ (thus, also $p^{S} = 1$), and $\tau^{\text{pay}} = \tau^{u}$, to have employment and payroll taxes equally shared between the employee and the employer. All prices are expressed in real terms at 2013 prices.

Beveridge and Wage Equations

I use the elasticity of matches to the unemployment rate $\beta = 0.5$ (Pissarides 1998) which is also within the range of estimates for the U.S. (Blanchard and Diamond 1989). At the benchmark equilibrium, the calibrated separation rate is approximately $\lambda = 3\%$, and the matching efficiency, $\mu = 0.67$. The calibrated separation rate is consistent with the es-

Table 2.1 : Benchmark assumptions.

Variables & Parameters	Notation	Value	Sources
Unemployment rate	u	0.06	BLS (2015c)
Vacancy rate	v	0.03	BLS (2015c)
Interest Rate	r	0.37	
Average wage rate	$\bar{w}$	1	
Elasticity of matches to unemployment rate	β	0.5	Blanchard and Diamond (1989)
Population	$\hat{N}$	1	
Intertemporal elasticity of substitution	ξ	0.25	Altig et al (2001)
Elasticity of substitution in production	ϵ	0.8	Altig et al (2001)
Production function capital share	ψ^Q	0.25	Altig et al (2001)
Depreciation rate	δ	0.08	Zodrow and Diamond (2013)
Depreciation rate for tax purposes	δ^τ	0.11	Zodrow and Diamond (2013)
Ratio debt-to-capital	α^D	0.3	Damodaran (2015) Zodrow and Diamond (2013)
Divident payout ratio	α^X	0.6	Damodaran (2015) Zodrow and Diamond (2013)
Unemployment insurance	ρ_1, ρ_2, ρ_3	0.79, 0.65, 0.49	IRS (2013)
	ρ_4, ρ_5, ρ_6	0.37, 0.28, 0.21	
	ρ_7, ρ_8, ρ_9	0.17, 0.13, 0.1	
	$\rho_{10}, \rho_{11}, \rho_{12}$	0.07, 0.06, 0.02	
Average tax on interest income	τ^r	0.18	IRS (2013)
Average employment/payroll tax	$\tau^u, \tau^{\text{pay}}$	0.08	IRS (2013)
Tax on interest income from assets	τ^{rA}	0.027	IRS (2013)
Tax on Social Security benefits	τ^M	0.2	IRS (2013)
Dividends tax	τ^X	0.24	CBO (2014)
Capital gains tax	τ^G	0.15	CBO (2014)
Property tax	τ^{pro}	0.02	Zodrow and Diamond (2013)

timates of Shimer (2005, 2012); and is somewhat lower than the 2001-2015 average of 3.4% (U.S. Department of Labor 2015d) which includes the effects of the 2008 economic downturn. The separation rate and matching efficiency are calibrated to obtain an unemployment rate, $u = 6\%$, which reflects the 1947-2015 average (U.S. Department of Labor 2015c). The vacancy rate is set to the 2001-2015 average, $v = 3\%$ (U.S. Department of Labor 2015d). Both unemployment and vacancy rate values imply an estimated probability of finding a job of $m/u = 48\%$, consistent with Shimer (2012). Furthermore, the resulting probability of filling a vacancy, $m/v = 95\%$ is consistent with Shimer (2005). The corre-

Table 2.2 : Benchmark calibrated variables.

Parameter	Notation	Value
Utility consumption share	ψ^C	0.91
Utility labor share	ψ^H	0.41
Intensity of consumption effects	η	-1.55
Intensity of labor supply effects	γ	0.25
Average cost per opened vacancy	c	0.4
Separation rate	λ	0.03
Matching efficiency	μ	0.67
Individual's bargaining power	$\sigma_1, \sigma_2, \sigma_3$	0.01,0.07,0.21
	$\sigma_4, \sigma_5, \sigma_6$	0.35, 0.49,0.58
	$\sigma_7, \sigma_8, \sigma_9$	0.67, 0.74, 0.79
	$\sigma_{10}, \sigma_{11}, \sigma_{12}$	0.86,0.90,0.97
Percentage of Social Sec. funded by Payroll tax	α^M	0.75
Government discretionary expenses	G	0.34

sponding matching rate, $m = 2.9\%$, also approximates the 2001-2015 average, 3.4% (U.S. Department of Labor 2015d).

Bargaining power parameters, σ_y, are estimated from the wage equation in the benchmark equilibrium. Their values ranges between 1 to 97 percent from the lowest to the highest income group. The average of all income groups is 0.56, close to the standard assumption of 0.5 in models that use a single representative individual (Pissarides 2000).

2.4.2 Individual Labor Supply

I assume that the individual labor supply averages eight hours a day (U.S. Department of Labor 2015b); that is, $\hat{H}^e = 2080$ hours per year per individual. Prices are set equal to one, that is $p = p^S = 1$. The elasticity of substitution of intertemporal consumption, $\xi = 0.25$ is set equal to its value in Altig et al. (2001), and falls in the range used in other CGE models. The utility consumption share, $\psi^C = 0.91$, and labor share, $\psi^H = 0.41$, are calibrated using the individual labor supply function, setting the average wage rate $\bar{w} = 1$ and the value of $\alpha^\psi = 0.56$ at the benchmark equilibrium.

Table 2.3 : Policy variables.

Variables	Notation	Benchmark	X	Flat
Percentage of tax for non-exempted goods	α^C	1	1	1
Interest tax inclusion	α^r	1	0	0
Interest expense deducted tax inclusion	α^d	1	0	0
Depreciation allowances inclusion	α^k	1	0	0
Investment expensed deduction	α^I	0	1	1
Dividend tax inclusion	α^x	1	0	0
Capital gains inclusion	α^g	1	0	0

The intensity of consumption effects, $\eta = -1.55$, and intensity of labor supply effects, $\gamma = 0.25$, were calibrated by setting the average uncompensated wage elasticity of labor supply $\Theta_{H,w} = 0.1$ and the average income elasticity of labor supply $\Theta_{H,\Omega} = -0.05$, values which are close to recent empirical estimates (McClelland and Mok 2012).

Job Creation Equation

At the benchmark equilibrium $\hat{L} = \sum_y \hat{N}_y H^e_y / \hat{H}^e = 1$, and $\bar{w} = 1$. The production function capital share is set to $\psi^Q = 0.25$ (Altig et al. 2001), and the elasticity of substitution of capital to $\epsilon = 0.8$. I use the optimal capital stock equation (equation 2.56) and the job creation equation (equation 2.70) to calibrate the average cost per open vacancy to $c = 0.4$.

Government

The percentage of Social Security funded with payroll taxes, $\alpha^M = 0.75$, is calibrated using equation 2.43, and government discretionary expenses $G = 0.34$ are calibrated such that equation A.78 is balanced. Table 2.3 shows the values of the α parameters that determine whether and the extent to which certain components are included in the tax base.

Table 2.4 : Aggregate Effects

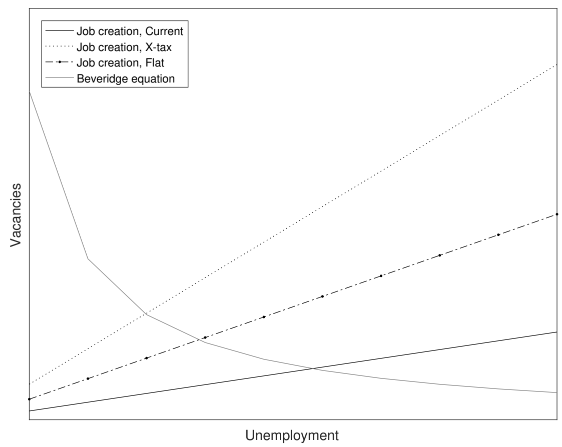
Variables	Notation	Benchmark	X	Flat
Unemployment rate	u	6.00%	3.11%	3.88%
Vacancy rate	v	3.00%	6.12%	4.85%
Market tightness	θ	0.50	1.96	1.25
Interest rate	r	0.37	0.29	0.31
Production	Q	1.42	1.51	1.38
Capital	K	0.53	0.32	0.48
Labor supply	L	1.00	0.97	0.95
Average wage	$\bar{w}$	1.00	1.02	0.98
Effective labor demand	eL	0.93	0.99	0.91
Marginal labor cost	ϕ^L	1.09	1.13	1.08
Marginal capital cost	ϕ^K	0.16	0.72	0.23
Marginal investment cost	phi^I	1.10	1.00	1.00
Rents from a filled vacancy	Π^f	0.14	0.27	0.23
Labor productivity	$\partial Q/\partial eN$	1.17	1.21	1.11
Marginal change in rents a from filled vacancy due to changes in the average wage	$\partial\Pi^f/\partial\bar{w}$	-1.39	-1.63	-1.69

2.4.3 Results

2.4.4 Aggregate variables

Labor market Both, the flat tax and the X-tax reforms, result in significant reductions in unemployment (table 2.4). This is caused by an important increase in market tightness ($\theta = v/u$) in both cases; from 0.5 to 1.25 with the flat tax, and to almost 2 with the X-tax (table 2.4). The increase in market tightness, depicted in figure 2.1, is reflected as a steeper slope of the job creation curve. Given that the X-tax reform leads to the largest market tightness, it also increases the probability of being hired more than the flat tax. The probability of being hired increases from 48 to 94 percent under the X-tax reform, compared to the (also high) increase to 75 percent probability under the flat tax. Consequently, the unemployment rate is lower under the X-tax, 3.1% percent, than under the flat tax, 3.9%. The probability of filling a vacancy falls from 95 to 48 and 60 percent with the flat tax and X-tax, respectively.

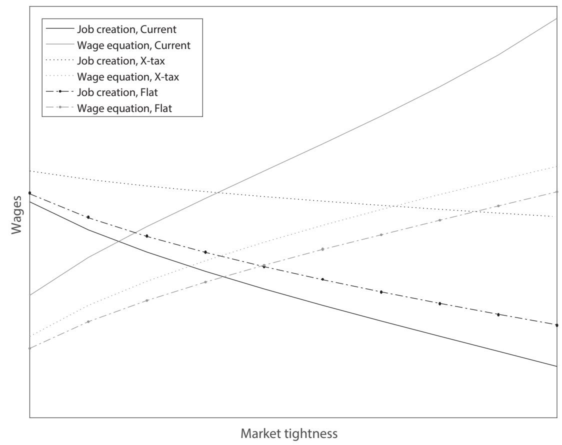
Figure 2.1 : Beveridge and Job Creation Curves



Given that the probability of filling a vacancy falls, it takes longer to fill the vacancy with a match. Thus, it becomes more expensive to keep a vacancy open under both reforms.

The increases in market tightness under both reforms and the corresponding reductions in unemployment are explained by a relative reduction in the cost of labor compared with the rents obtained from a filled vacancy. Both reforms raise the marginal loss in rents from a filled vacancy due to an increase in the average wage ($\partial\Pi^f/\partial\bar{w}$). In the case of the X-tax $\partial\Pi^f/\partial\bar{w}$ increases by 17 percent, while under the flat tax it increases by 21 percent (table 2.4). Hence, it becomes more expensive to raise wages under the X-tax or flat tax compared with the benchmark equilibrium. As a result, there is an increase in the incentive to reduce wages from the firm's side. This effect is represented by downward shifts of the wage curves under both reforms in figure 2.2. There is also an opposite effect, which increases (for almost all income groups) their incentive to bargain for higher wages, as explained in section concerning individuals' effects. However, the downward effect due to the increase in the average wage prevails.

Figure 2.2 : Job Creation and Wage curves.

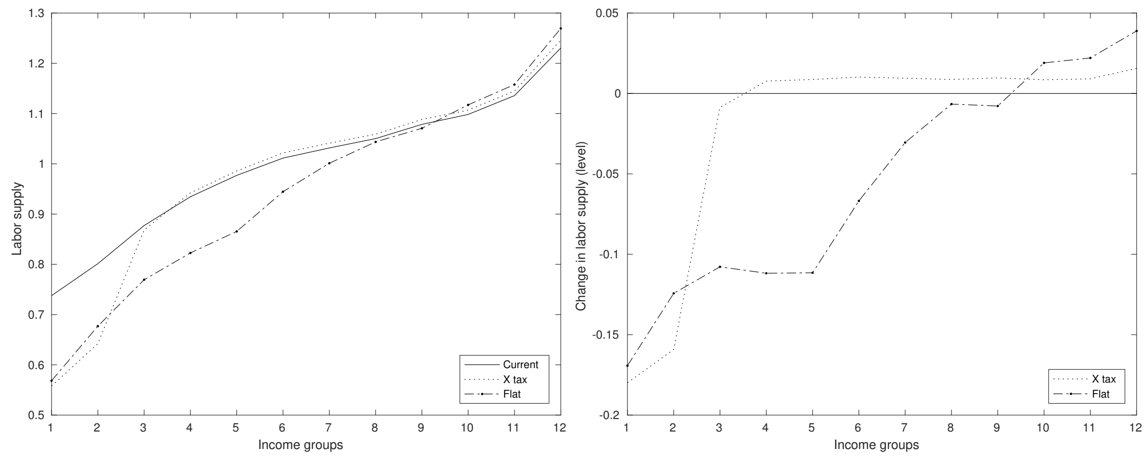


Additionally, the rents from a vacancy filled (Π^F) increase in both cases, from 0.14 to 0.27 under the X-tax, and to 0.23 under the flat tax. It shows that benefits of the reduction in wage costs (flat tax) or increased labor productivity (X-tax) exceeds the cost of opening more vacancies. The marginal labor cost, ϕ^L, reduces by 1 percent with the flat tax but increases almost 4 percent with the X-tax. The increase in marginal labor costs with the X-tax is offset with an increase the labor productivity of approximately 3 percent. This is partially due to shifting from capital towards labor given the increase in the cost of capital

Table 2.5 : Taxes.

Variables	Notation	Benchmark	X	Flat
Average income tax	$\bar{\tau}_1$, $\bar{\tau}_2$, $\bar{\tau}_3$	0.10, 0.10, 0.10	0.10, 0.10, 0.10	0.31
	$\bar{\tau}_4$, $\bar{\tau}_5$, $\bar{\tau}_6$,	0.10, 0.10, 0.11	0.10, 0.10, 0.11	
	$\bar{\tau}_7$, $\bar{\tau}_8$, $\bar{\tau}_9$	0.12, 0.13, 0.15	0.12, 0.13, 0.15	
	$\bar{\tau}_{10}$, $\bar{\tau}_{11}$, $\bar{\tau}_{12}$	0.17, 0.19, 0.33	0.17, 0.19, 0.34	
Marginal income tax	τ_1^w, τ_2^w, τ_3^w	0.10, 0.10, 0.10	0.10, 0.10, 0.10	0.31
	τ_4^w, τ_5^w, τ_6^w	0.13, 0.15, 0.15	0.13, 0.15, 0.15	
	τ_7^w, τ_8^w, τ_9^w	0.18, 0.21, 0.21	0.19, 0.22, 0.22	
	τ_{10}^w, τ_{11}^w, τ_{12}^w	0.26, 0.27, 0.38	0.27, 0.28, 0.40	
Sales	τ^{con}	0.08	0	0
Business income tax	τ^{π}	0.35	0.40	0.31

Figure 2.3 : Labor supply, and changes in labor supply (level).



stock, ϕ^K (table 2.4). Also, the increased cost of filling a vacancy (it takes longer to fill it with a match) is balanced with lower interest rates in both reforms, which reduce the cost of investing in those vacancies (right-hand side of equation 2.70). Figure 2.2 shows the upward shifts of the X-tax and flat tax job creation curves. This effect pushes wages up, compensating for the downward effect that occurs under the wage equation. In the end, the downward effect on wages dominates under the flat tax, in which case the average wage falls by 2 percent, while the upward effect dominates under the X-tax, increasing the average wage, also by 2 percent.

A standard result under a consumption tax reform is the substitution effect of the wage change on the choice between consumption and leisure, which in principle could reduce the incentive to work. Aggregate labor supply falls 3 percent under the X-tax, while it drops 5 percent with the flat tax. This is due in part to maintaining the same average income tax under the X-tax, but increasing the average income tax rate for most of the income groups with the flat tax.

Output and capital One of the advantages of a consumption tax is its positive effect on output and capital accumulation (Altig et al. 2001). Capital stock falls from 0.53 to 0.32 under the X-tax, and to 0.48 with the flat tax, respectively. Part of this reduction is due to an increase in the after-tax marginal cost of capital under both reforms, which exceeds the benefits obtained from investment expensing. That is, while ϕ^I falls from 1.10 to 1 in both reforms, ϕ^K raises from 0.16 to 0.23 under the flat tax, and to 0.72 under the X-tax (table 2.4). In the case of the X-tax, the business tax rate increases to match the income tax rate applied to the highest income group (table 2.5). Combined with the broadening of the business tax base –by the elimination of interest expenses deductions– reduces the capital stock by a significant amount. In the case of the flat tax, the lower business tax rate compensates the negative effect on the capital stock, thus reducing it in a much lower magnitude. Additionally, in this model, firms not only invest in capital (K) but also in effective labor through the number of vacancies opened. Thus, more vacancies lead to less capital invested, as is shown in table 2.4. This result is different from those obtained in the standard CGE literature. Output increases under the X-tax (table 2.4) due to an increase in the effective labor demand, eL. However, it decreases under the flat-tax given that the average tax rate increases for almost all income groups, thus reducing the aggregate demand for goods.

Individuals

Hours, wages, savings and consumption by income group The flat tax increases the average tax rate to 31 percent for all the income groups except the top one. In contrast, the X-tax average income tax is kept roughly the same as in the benchmark equilibrium (table 2.5), which plays a significant role in the different responses of individual labor supply of the various income groups. In an X-tax reform, almost all income groups (except the first

Figure 2.4 : Wages, changes in level (left) and percentages (right).

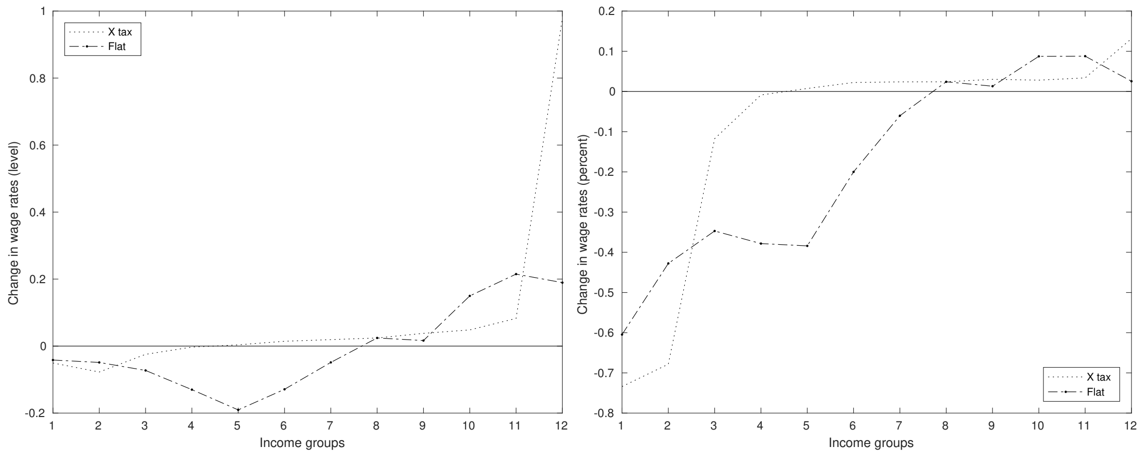
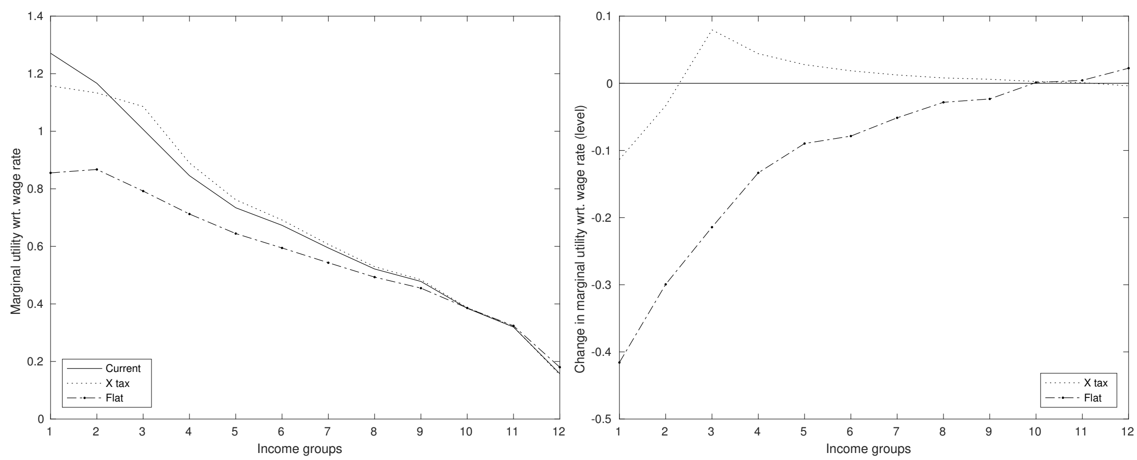


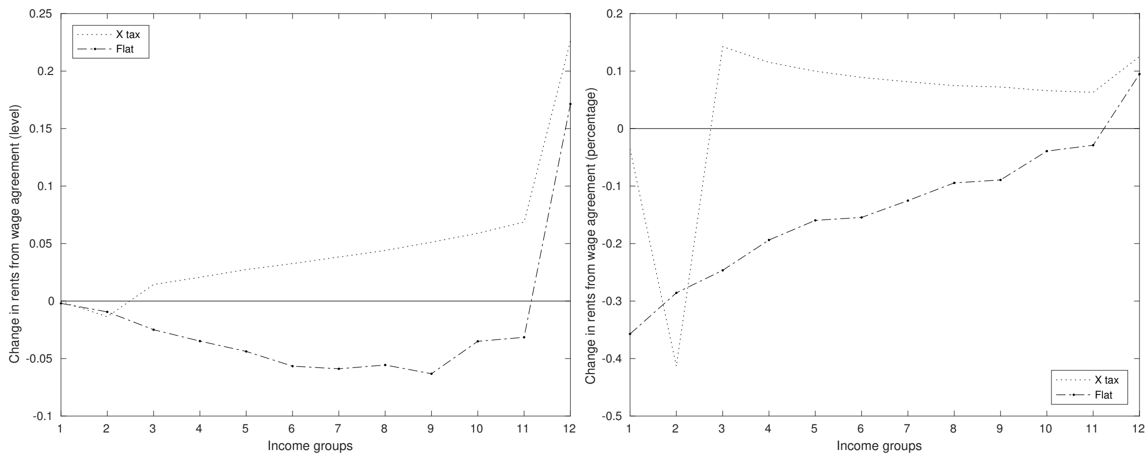
Figure 2.5 : Marginal utility from wage rate and changes in level.



two) change their hours of work slightly (figure 2.3). By comparison, the flat tax has a marked reduction for the first seven income groups, and increase for the last three.

For both reforms, the changes in hours of work follow a similar pattern to proportional changes in the wage rate (figure 2.4). In the case of the X-tax, the marginal utility obtained from a change in the wage rate ($\partial U_y^e/\partial w_y$) is, on average, kept almost the same with respect to the benchmark equilibrium (figure 2.5). However, $\partial U_y^e/\partial w_y$ reduces for the first two income groups, and increases for income groups 3 and 4. Also, the individual rents obtained

Figure 2.6 : Individual rents from wage agreement, change in levels (left) and percentages (right).

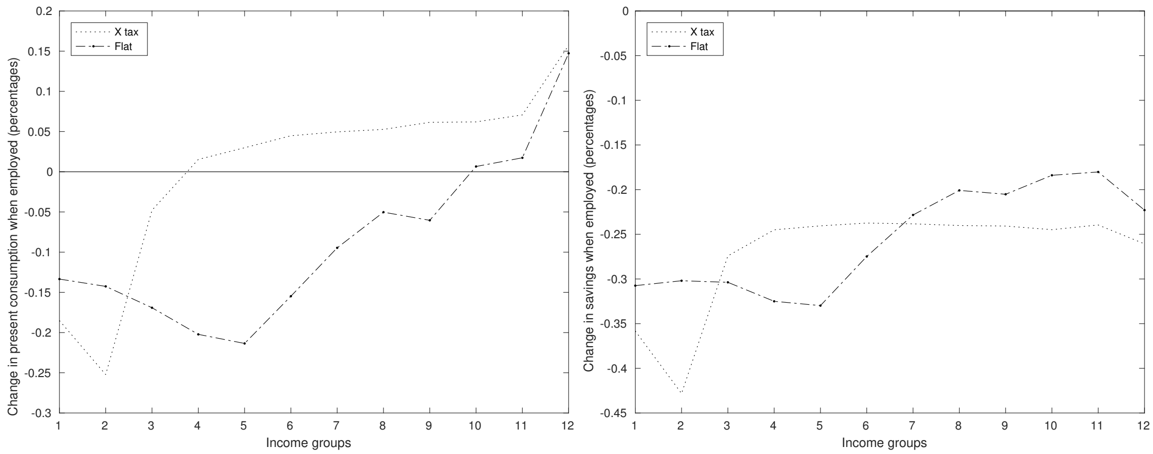


from wage agreement ($U_y^e - U_y^u$) increase for almost all income groups (figure 2.6). Thus, given the growth of individual rents from wage agreement, there are incentives to bargain for higher wages. However, the incentive to reduce wages by the firms counters the increase of individual rents. Therefore, there is only a small raise of wages for those income groups that increased their individual rents and had large enough bargaining power.

On the other hand, the flat tax decreases the marginal utility from wages for income groups 1 to 9, thus reducing their incentive to bargain for a higher wage. However, their individual rents from wage agreement also reduce, which has the opposite effect of increasing their incentive to bargain. The combined result leads to a raise in wages for income groups 10 to 12, and almost no change for groups 8 and 9. This result is also reflected in their hours of work.

Present consumption falls under the flat tax, on average, except for income groups 10 to 12 who increase their consumption. In the case of the X-tax, present consumption increases except for income groups 1 to 3. Savings are affected almost equally among income groups under the X-tax, about 25 percent on average, except for the lowest income decile. Under

Figure 2.7 : Changes in percentages of present consumption (left) and savings (right).



the flat tax, savings fall by 30 on average for income groups 1 to 5, 27 percent for income group 6, and by 20 percent on average for income groups 7 to 12. The main reason behind the decline in savings under both reforms is the interest rate reduction.

Incidence Figure 2.8 shows the incidence of tax reforms measured by their amount of equivalent variation. On average, both reforms have some negative impact on the welfare of the income groups. The X-tax reform is slightly more neutral to income groups 1 to 7 when the individuals are employed, and 1 to 5 when they are unemployed.

In contrast, the flat tax is more neutral to income groups 10 and 11 when they are employed, and 6 to 11 when they are unemployed. The top 2 percent either enjoys the largest benefit with the flat tax, or the suffers the highest loss with the X-tax. Between both reforms, the X-tax is less regressive.

2.5 Conclusions

I simulated the effects of the enactment of a Hall-Rabushka flat tax and the Bradford X-tax. I found that replacing the current progressive income tax system either with an X tax

Figure 2.8 : Index of equivalent variation when employed (left), and unemployed (right).

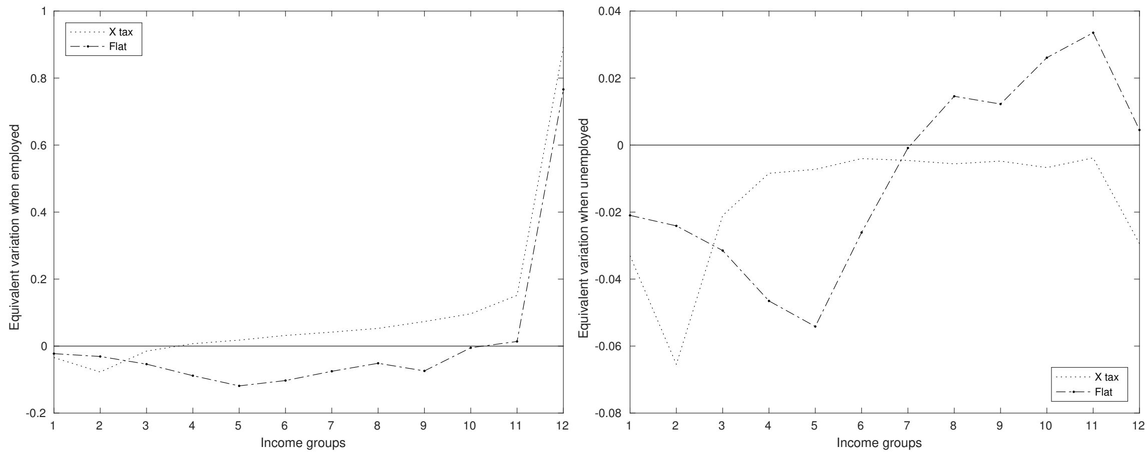
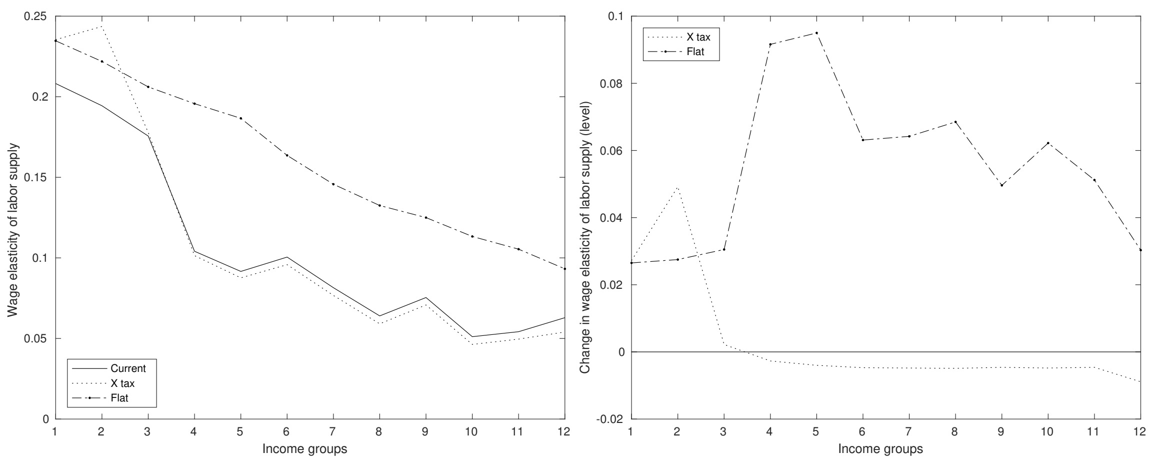


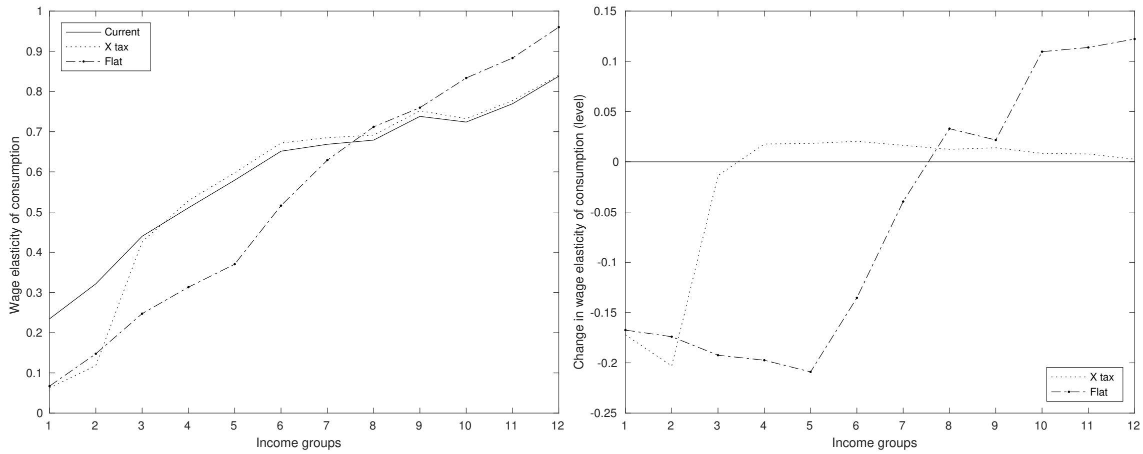
Figure 2.9 : Wage elasticities of labor supply and changes.



or a flat tax reform lead to a decrease in unemployment. Both reforms reduce the cost of labor and encourage firms to increase the number of vacancies. Hence, the market tightness increases and the unemployment rate drops. Output and labor supply remain practically the same, while capital falls from its levels from the benchmark equilibrium with both reforms.

Both reforms reduce the welfare of most income groups. However, the enactment of a flat tax, which reduces the degree of tax progressivity compared with the X tax, affects dis-

Figure 2.10 : Consumption elasticities of labor supply and changes.



proportionately the wage of lower income groups, which affects their consumption levels. Measured by its equivalent variation, the flat tax reform is more regressive than the X-tax.

These simulations show how increases in growth, capital accumulation and economic efficiency that typically occur under standard CGE analyses could be reduced, while the efficiency in the labor market is increased through unemployment reduction.

Appendix A

A.1 General Equilibrium

A.1.1 Goods Market

Individuals

Employed Individuals I denote the Lagrangian function as $\mathcal{L}$, and the Lagrange multiplier as χ. In the case the individual is employed, it is defined as

$$\mathcal{L} = \frac{[C_y]^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} - \psi^H \frac{[H_y]^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} + \chi \left[W_y^\tau + \Omega_y - E_y\right] \tag{A.1}$$

where

$$C_y = \left[[\psi^C]^{\frac{1}{\xi}}[C_y^P]^{\frac{\xi-1}{\xi}} + [1-\psi^C]^{\frac{1}{\xi}}[C_y^F]^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}, \tag{A.2}$$

$$W_y^\tau = w_y H_y - [\bar{\tau}_y + \tau_y^{\text{pay}}]B_y^e, \tag{A.3}$$

$$B_y^e = w_y H_y - \omega^{\text{ded}}, \tag{A.4}$$

$$\Omega_y = [1-\tau^M]M_y + [1-\alpha^r\tau^r]rA_y, \tag{A.5}$$

$$E_y = [1+\tau^C]pC_y^P + \frac{[1+\tau^C]pC_y^F}{[1-\alpha^r\tau^r]r^S}. \tag{A.6}$$

The optimal values for individual labor supply and present and future consumption are denoted by H_y^e, C_y^{Pe}, and C_y^{Fe}, respectively. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_y^P} = [C_y(C_y^{Pe}, C_y^{Fe})]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^P}(C_y^{Pe}, C_y^{Fe}) - \chi[1+\tau^C]p = 0 \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial C_y^F} = [C_y(C_y^{Pe}, C_y^{Fe})]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^F}(C_y^{Pe}, C_y^{Fe}) - \chi \frac{[1+\tau^C]p}{[1-\alpha^r\tau^r]r^S} = 0 \tag{A.8}$$

$$\frac{\partial \mathcal{L}}{\partial H_y} = -\psi_y^H [H_y^e]^{\frac{1}{\gamma}} + \chi[1 - \tau_y^w(H_y^e) - \tau_y^{\text{pay}}]w_y = 0 \tag{A.9}$$

$$\frac{\partial \mathcal{L}}{\partial \chi} = W_y^\tau(H_y^e) + \Omega_y - E_y(C_y^{Pe}, C_y^{Fe}) = 0 \tag{A.10}$$

Consumption Dividing (A.7) by (A.8) yields

$$\left[\frac{\psi^C C_y^{Fe}}{[1-\psi^C]C_y^{Pe}}\right]^{\frac{1}{\xi}} = [1-\alpha^r\tau^r]r^S.$$

That is,

$$\frac{C_y^{Fe}}{C_y^{Pe}} = \left[[1-\alpha^r\tau^r]r^S\right]^\xi \left[\frac{1-\psi^C}{\psi^C}\right]. \tag{A.11}$$

Plugging equation A.11 into the expenditures equation, A.6, leads to

$$\begin{aligned} E_y(C_y^{Pe}, C_y^{Fe}) &= [1+\tau^C]pC_y^{Pe} + \frac{[1+\tau^C]p}{[1-\alpha^r\tau_y^r]r^S}\left[[1-\alpha^r\tau^r]r^S\right]^\xi \left[\frac{1-\psi^C}{\psi^C}\right] C_y^{Pe} \\ &= \frac{[1+\tau^C]pC_y^{Pe}}{\alpha^\psi}, \end{aligned} \tag{A.12}$$

where

$$\alpha^\psi = \frac{\psi^C}{\psi^C + [1-\psi^C]\left[\frac{1}{[1-\alpha^r\tau_y^r]r^S}\right]^{1-\xi}}. \tag{A.13}$$

Using equation A.12 in the budget constraint, equation A.10, I obtain

$$C_y^{Pe} = \alpha^{\psi} \frac{W_y^{\tau} + \Omega_y}{[1+\tau^C]p}. \tag{A.14}$$

Equation A.14 along with equation A.11 lead to

$$C_y^{Fe} = [1-\alpha^{\psi}] \frac{[1-\alpha^r \tau^r] r^S [W_y^{\tau} + \Omega_y]}{[1+\tau^C]p} \tag{A.15}$$

Thus, the optimal consumption utility function, $C_y^e = C_y(C_y^{Pe}, C_y^{Fe})$, is

$$\begin{aligned} C_y^e &= \left[[\psi^C]^{\frac{1}{\xi}} \left[\frac{\alpha^{\psi}[W_y^{\tau} + \Omega_y]}{[1+\tau^C]p} \right]^{\frac{\xi-1}{\xi}} + [1-\psi^C]^{\frac{1}{\xi}} \left[\frac{[1-\alpha^{\psi}][W_y^{\tau} + \Omega_y]}{\frac{[1+\tau^C]p}{[1-\alpha^r\tau^r]r^S}} \right]^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}} \\ &= \frac{W_y^{\tau} + \Omega_y}{[1+\tau^C]p} \left[[\psi^C]^{\frac{1}{\xi}} \left[\alpha^{\psi}\right]^{\frac{\xi-1}{\xi}} + [1-\psi^C]^{\frac{1}{\xi}} \left[[1-\alpha^r\tau^r] r^S [1-\alpha^{\psi}] \right]^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1.}} \end{aligned}$$

Using the value of α^{ψ}, equation A.13, it becomes

$$\begin{aligned} C_y^e &= \frac{W_y^{\tau} + \Omega_y}{[1+\tau^C]p} \left[[\psi^C]^{\frac{1}{\xi}} \left[\frac{\psi^C}{\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}} \right]^{\frac{\xi-1}{\xi}} \right. \\ &\left. + [1-\psi^C]^{\frac{1}{\xi}} \left[[1-\alpha^r\tau_y^r] r^S \frac{[1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}}{\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}} \right]^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}, \\ &= \frac{W_y^{\tau} + \Omega_y}{[1+\tau^C]p} \left[\frac{\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}}{\left[\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}\right]^{\frac{\xi-1}{\xi}}} \right]^{\frac{\xi}{\xi-1}}, \\ &= \frac{W_y^{\tau} + \Omega_y}{[1+\tau^C]p} \left[\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1} \right]^{\frac{1}{\xi-1}}. \end{aligned} \tag{A.16}$$

That is,

$$C_y^e = \frac{W_y^\tau(H_y^e) + \Omega_y}{[1+\tau^C]p} \left[\frac{\alpha^\psi}{\psi^C}\right]^{\frac{1}{1-\xi}}. \tag{A.17}$$

Savings Since

$$p^S S_y^e = \frac{[1+\tau^C]pC_y^{Fe}}{[1-\alpha^r\tau^r]r^S},$$

by replacing the future consumption equation, A.15, individual savings are equal to

$$S_y^e = \frac{[1-\alpha^\psi][W_y^\tau + \Omega_y]}{p^S} = \left[\frac{r^S}{p^\kappa \kappa}\right][1-\alpha^\psi][W_y^\tau + \Omega_y]. \tag{A.18}$$

Individual labor supply To obtain the individual labor supply function, H_y^e, I first divide (A.7) by (A.9), such that

$$\frac{[C_y^e]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^P}(C_y^{Pe}, C_y^{Fe})}{\psi_y^H [H_y^e]^{\frac{1}{\gamma}}} = \frac{[1+\tau^C]p}{[1-\tau_y^w(H_y^e) - \tau_y^{\text{pay}}]w_y}. \tag{A.19}$$

Rearranging the terms, the individual labor supply function is equal to

$$\psi_y^H [H_y^e]^{\frac{1}{\gamma}} = \frac{[C_y^e]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^P}[1-\tau_y^w(H_y^e) - \tau_y^{\text{pay}}]w_y}{[1+\tau^C]p}, \text{ or}$$

$$H_y^e = \left[\frac{[C_y^e]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^P}[1-\tau_y^w(H_y^e) - \tau_y^{\text{pay}}]w_y}{\psi_y^H[1+\tau^C]p}\right]^\gamma. \tag{A.20}$$

The value of $\frac{\partial C_y}{\partial C_y^P}(C_y^{Pe}, C_y^{Fe})$ is equal to

$$\frac{\partial C_y}{\partial C_y^P} = \left[[\psi^C]^{\frac{1}{\xi}}[C_y^{Pe}]^{\frac{\xi-1}{\xi}} + [1-\psi^C]^{\frac{1}{\xi}}[C_y^{Fe}]^{\frac{\xi-1}{\xi}}\right]^{\frac{1}{\xi-1}} [\psi^C]^{\frac{1}{\xi}}[C_y^{Pe}]^{-\frac{1}{\xi}}$$
$$= \left[\psi^C + \psi^C\left[\frac{1-\psi^C}{\psi^C}\right]^{\frac{1}{\xi}}\left[\frac{C_y^{Fe}}{C_y^{Pe}}\right]^{\frac{\xi-1}{\xi}}\right]^{\frac{1}{\xi-1}}.$$

By using the value of $\frac{C_y^{Fe}}{C_y^{Pe}}$ (equation A.11),

$$\frac{\partial C_y}{\partial C_y^P} = \left[\psi^C + \psi^C\left[\frac{1-\psi^C}{\psi^C}\right]^{\frac{1}{\xi}}\left[\left[[1-\alpha^r\tau^r]r^S\right]^{\xi}\left[\frac{1-\psi^C}{\psi^C}\right]\right]^{\frac{\xi-1}{\xi}}\right]^{\frac{1}{\xi-1}}$$
$$= \left[\psi^C + [1-\psi^C]\left[[1-\alpha^r\tau^r]r^S\right]^{\xi-1}\right]^{\frac{1}{\xi-1}} = \left[\frac{\psi^C}{\alpha^\psi}\right]^{\frac{1}{\xi-1}}. \tag{A.21}$$

Finally, by replacing the value of C_y^e (equation A.17) and $\frac{\partial C_y}{\partial C_y^P}$ (equation A.21) into equation A.20, it leads to

$$H_y^e = \left[\frac{W_y^\tau(H_y^e) + \Omega_y}{[1+\tau^C]p}\right]^{\frac{\gamma}{\eta}} \left[\frac{\psi^C}{\alpha^\psi}\right]^{\frac{\gamma[1+\eta]}{\eta[\xi-1]}} \left[\frac{[1-\tau_y^w(H_y^e) - \tau_y^{\text{pay}}]w_y}{\psi_y^H[1+\tau^C]p}\right]^{\gamma}. \tag{A.22}$$

Elasticities Here I calculate the wage and income elasticities of labor supply and the wage elasticity of consumption, used in the wage equation and calibration process. For the wage elasticity of labor supply, I first take the implicit derivative of equation A.22 with respect to w_y. Since

$$\frac{\partial W_y^\tau}{\partial w_y} = \frac{\partial w_y H_y^e}{\partial w_y} - [\tau_y^w + \tau_y^{\text{pay}}]\frac{\partial B_y^e}{\partial w_y} = [1-\tau_y^w - \tau_y^{\text{pay}}]\left[H_y^e + w_y\frac{\partial H_y^e}{\partial w_y}\right], \tag{A.23}$$

$$\frac{\partial H_y^e}{\partial w_y} = H_y^e \gamma \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y}{\psi_y^H[1+\tau^C]p}\right]^{-1} \left[\frac{1-\tau_y^w - \tau_y^{\text{pay}} - w_y \alpha_y^w \left[H_y^e + w_y \frac{\partial H_y}{\partial w_y}\right]}{\psi_y^H[1+\tau^C]p}\right]$$

$$+ H_y^e \frac{\gamma}{\eta} \left[\frac{W_y^\tau + \Omega_y}{[1+\tau^C]p}\right]^{-1} \frac{1-\tau_y^w - \tau_y^{\text{pay}}}{[1+\tau^C]p} \left[H_y^e + w_y \frac{\partial H_y^e}{\partial w_y}\right]$$

$$= H_y^e \gamma \left[\frac{1}{w_y} - \frac{\alpha_y^w \left[H_y^e + w_y \frac{\partial H_y}{\partial w_y}\right]}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] + H_y^e \frac{\gamma}{\eta} \left[\frac{1-\tau_y^w - \tau_y^{\text{pay}}}{W_y^\tau + \Omega_y}\right] \left[H_y^e + w_y \frac{\partial H_y^e}{\partial w_y}\right].$$

Thus,

$$\frac{\partial H_y^e}{\partial w_y} \left[1 + H_y^e \gamma \frac{\alpha_y^w w_y}{1-\tau_y^w - \tau_y^{\text{pay}}} - H_y^e \frac{\gamma}{\eta} \frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y}{W_y^\tau + \Omega_y}\right]$$
$$= H_y^e \gamma \left[\frac{1}{w_y} - \frac{\alpha_y^w H_y^e}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] + H_y^e \frac{\gamma}{\eta} \left[\frac{1-\tau_y^w - \tau_y^{\text{pay}}}{W_y^\tau + \Omega_y}\right] H_y^e,$$

and

$$\frac{\partial H_y^e}{\partial w_y} = H_y^e \frac{\gamma \left[\frac{1}{w_y} - \frac{\alpha_y^w H_y^e}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] + \frac{\gamma}{\eta} \left[\frac{1-\tau_y^w - \tau_y^{\text{pay}}}{W_y^\tau + \Omega_y}\right] H_y^e}{1 + H_y^e \gamma \left[\frac{\alpha_y^w w_y}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] - H_y^e \frac{\gamma}{\eta} \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y}{W_y^\tau + \Omega_y}\right]}.$$

As a result, the wage elasticity of labor supply, Θ_{Hw}, is equal to

$$\Theta_{Hw} = \frac{\partial H_y^e}{\partial w_y} \frac{w_y}{H_y^e} = \frac{\gamma \left[1 - \frac{\alpha_y^w w_y H_y^e}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] + \frac{\gamma}{\eta} \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y H_y^w}{W_y^\tau + \Omega_y}\right]}{1 + \gamma \left[\frac{\alpha_y^w w_y H_y^e}{1-\tau_y^w - \tau_y^{\text{pay}}}\right] - \frac{\gamma}{\eta} \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}]w_y H_y^e}{W_y^\tau + \Omega_y}\right]}. \tag{A.24}$$

Note that in the case of $\alpha_y^w = 0$ (the case with the flat tax) and $\eta \to \infty$, the wage elasticity of labor supply is equal to γ.

Similarly, since

$$\frac{\partial W_y^\tau}{\partial \Omega_y} = [1-\tau_y^w - \tau_y^{\text{pay}}] \frac{\partial B_y^e}{\partial \Omega_y} = [1-\tau_y^w - \tau_y^{\text{pay}}] w_y \frac{\partial H_y^e}{\partial \Omega_y},$$

the implicit derivative of the labor supply with respect to non-wage income is such that

$$\frac{\partial H_y^e}{\partial \Omega} = \frac{\gamma}{\eta} H_y^e \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y \frac{\partial H_y}{\partial \Omega} + 1}{W_y^\tau + \Omega_y} \right] - \gamma H_y^e \left[\frac{\alpha^w w_y \frac{\partial H_y}{\partial \Omega}}{1-\tau_y^w - \tau_y^{\text{pay}}} \right].$$

Hence,

$$\frac{\partial H_y^e}{\partial \Omega} \left[1 - \frac{\gamma}{\eta} H_y^e \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y}{W_y^\tau + \Omega_y} \right] + \gamma H_y^e \left[\frac{\alpha^w w_y}{1-\tau_y^w - \tau_y^{\text{pay}}} \right] \right] = \frac{\gamma}{\eta} H_y^e \left[\frac{1}{W_y^\tau + \Omega_y} \right],$$

and

$$\frac{\partial H_y^e}{\partial \Omega} = \frac{\frac{\gamma}{\eta} H_y^e \left[\frac{1}{W_y^\tau + \Omega_y} \right]}{1 - \frac{\gamma}{\eta} H_y^e \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y}{W_y^\tau + \Omega_y} \right] + \gamma H_y^e \left[\frac{\alpha^w w_y}{1-\tau_y^w - \tau_y^{\text{pay}}} \right]}.$$

This leads to an income elasticity of labor supply, $\Theta_{H\Omega}$, equal to

$$\Theta_{H\Omega} = \frac{\partial H_y^e}{\partial \Omega} \frac{\Omega}{H_y^e} = \frac{\frac{\gamma}{\eta} \left[\frac{\Omega}{W_y^\tau + \Omega_y} \right]}{1 + \gamma \left[\frac{\alpha^w w_y H_y^e}{1-\tau_y^w - \tau_y^{\text{pay}}} \right] - \frac{\gamma}{\eta} \left[\frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y H_y^e}{W_y^\tau + \Omega_y} \right]} \tag{A.25}$$

Finally, to calculate the wage elasticity of consumption, Θ_{Cw}, the implicit derivative $\frac{\partial C_y^e}{\partial w_y}$ is

$$\frac{\partial C_y^e}{\partial w_y} = \frac{1-\tau_y^w - \tau_y^{\text{pay}}}{[1+\tau^C] p} \left[H_y^e + w_y \frac{\partial H_y^e}{\partial w_y} \right] \left[\frac{\psi^C}{\alpha^\psi} \right]^{\frac{1}{\xi - 1}},$$

which leads to

$$\Theta_{Cw} = \frac{\partial C_y^e}{\partial w_y} \frac{w_y}{C_y^e} = \frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y}{W_y^\tau + \Omega_y} \left[H_y^e + w_y \frac{H_y^e}{w_y} \frac{\partial H_y^e}{\partial w_y} \frac{w_y}{H_y^e} \right], \text{ or}$$

$$\Theta_{Cw} = \frac{\partial C_y^e}{\partial w_y} \frac{w_y}{C_y^e} = \frac{[1-\tau_y^w - \tau_y^{\text{pay}}] w_y H_y^e}{W_y^\tau + \Omega_y} \left[1 + \Theta_{Hw} \right]; \tag{A.26}$$

Unemployed individuals When the individual is unemployed, the Lagrangian is equal to

$$\mathcal{L} = \frac{[C_y]^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \chi\left[P_y^\tau + \Omega_y - E_y\right], \tag{A.27}$$

where

$$P_y^\tau = \rho_y w_y H_y - \bar{\tau}_y B_y^u, \tag{A.28}$$

$$B_y^u = \rho_y w_y H_y - \omega^{\text{ded}}. \tag{A.29}$$

The optimal values for present and future consumption are denoted by C_y^{Pu} and C_y^{Fu}, respectively. The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_y^P} = [C_y(C_y^{Pu}, C_y^{Fu})]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^P}(C_y^{Pu}, C_y^{Fu}) - \chi[1+\tau^C]p = 0 \tag{A.30}$$

$$\frac{\partial \mathcal{L}}{\partial C_y^F} = [C_y(C_y^{Pu}, C_y^{Fu})]^{\frac{1}{\eta}} \frac{\partial C_y}{\partial C_y^F}(C_y^{Pu}, C_y^{Fu}) - \chi\frac{[1+\tau^C]p}{[1-\alpha^r\tau^r]r^S} = 0 \tag{A.31}$$

$$\frac{\partial \mathcal{L}}{\partial \chi} = P_y^\tau + \Omega_y - E_y(C_y^{Pu}, C_y^{Fu}) = 0 \tag{A.32}$$

Consumption Dividing (A.30) with (A.31) yields

$$\frac{C_y^{Fu}}{C_y^{Pu}} = \left[[1-\alpha^r\tau^r]r^S\right]^\xi \left[\frac{1-\psi^C}{\psi^C}\right]. \tag{A.33}$$

Using this equation (A.33), the expenditures equation (A.6) becomes

$$E(C_y^{Pu}, C_y^{Fu}) = \frac{[1+\tau^C]pC_y^{Pu}}{\alpha^\psi}. \tag{A.34}$$

Replacing equation A.34 in the budget constraint, equation A.32, I obtain

$$C_y^{Pu} = \frac{\alpha^{\psi}[P_y^{\tau} + \Omega_y]}{[1+\tau^C]p}. \tag{A.35}$$

Equation A.35 along with equation A.11 lead to

$$C_y^{Fu} = [1-\alpha^{\psi}]\frac{[1-\alpha^r\tau^r]r^S[P_y^{\tau} + \Omega_y]}{[1+\tau^C]p}. \tag{A.36}$$

From equations A.35 and A.36, the optimal consumption function when the individual is unemployed, $C_y^u = C_y(C_y^{Pu}, C_y^{Fu})$ is equal to

$$C_y^u = \frac{P_y^{\tau}(H_y^e) + \Omega_y}{[1+\tau^C]p}\left[\frac{\alpha^{\psi}}{\psi^C}\right]^{\frac{1}{1-\xi}}. \tag{A.37}$$

Savings Individual savings are equal to

$$S_y^u = \frac{[1-\alpha^{\psi}][P_y^{\tau} + \Omega_y]}{p^S} = \left[\frac{r^S}{p^{\kappa}\kappa}\right][1-\alpha^{\psi}][P_y^{\tau} + \Omega_y]. \tag{A.38}$$

Firms

$$\Pi = \alpha^v\left[\alpha^{\pi}[\pi - \tau^{\pi}B^{\pi} - \delta K] + \delta K - I\right] \tag{A.39}$$

Firms solve the constrained maximization of their value Π subject to $I = \delta K$. The value pf Π is equal to

$$\Pi = \alpha^v \alpha^\pi \left[pQ - [1+\tau^u]\bar{w}eL - cvN - rD - \tau^{\text{pro}} K \right.$$
$$-\tau^\pi \left[pQ - \tau^{\text{pro}} K - [1+\tau^u]\bar{w}eL - cvN - \alpha^I I - \alpha^d rD - \alpha^k \delta^\tau K^\tau\right] - \delta K\Big]$$
$$+ \alpha^v \delta K - \alpha^v I \tag{A.40}$$

where,

$$\alpha^v = \frac{1-\alpha^g \tau^G}{[1-\alpha^\tau \tau^r] r} \tag{A.41}$$
$$\alpha^\pi = \frac{\alpha^X [1-\alpha^x \tau^X] + [1-\alpha^X][1-\alpha^g \tau^G]}{1-\alpha^g \tau^G}. \tag{A.42}$$

From the Beveridge equation (equation 2.35), $e = \mu\theta^{-\beta} v/\lambda$, the rate of employment in terms of vacancies. Thus,

$$\Pi = \alpha^v \alpha^\pi [1-\tau^\pi] pQ - \alpha^v \alpha^\pi [1-\tau^\pi] \left[[1+\tau^u]\bar{w}\frac{\mu\theta^{-\beta}}{\lambda}\frac{L}{N} + c\right] vN$$
$$- \left[\alpha^v \alpha^\pi [1-\tau^\pi]\tau^{\text{pro}} + \alpha^v \alpha^\pi [1-\alpha^d \tau^\pi] r\alpha^D + \alpha^v \alpha^\pi \delta - \alpha^v \delta\right] K$$
$$- \left[\alpha^v \alpha^\pi \tau^\pi [-\alpha^I - \alpha^k] + \alpha^v\right] I \tag{A.43}$$

with

$$Q = \left[[\psi^Q]^{\frac{1}{\epsilon}} K^{1-\frac{1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}} [eL]^{1-\frac{1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{A.44}$$

Denote

$$\phi^I = \frac{1}{1-\tau^\pi}\left[\tau^\pi[-\alpha^I - \alpha^k] + \frac{1}{\alpha^\pi}\right], \tag{A.45}$$

$$\phi^K = \tau^{\text{pro}} + \frac{1-\alpha^d\tau^\pi}{1-\tau^\pi} r\alpha^D + \frac{1}{1-\tau^\pi}\delta - \frac{1}{\alpha^\pi[1-\tau^\pi]}\delta, \text{ and} \tag{A.46}$$

$$\phi^L = \frac{\lambda}{\mu\theta^{-\beta}}\frac{cN}{L} + [1+\tau^u]\bar{w}. \tag{A.47}$$

These are the after-tax marginal cost of each factor of production: ϕ^K is the marginal cost per additional unit of capital, and ϕ^L is the marginal cost per additional hour of employed labor. Then, the first order conditions are:

$$\frac{\partial \Pi}{\partial v} = \alpha^v\alpha^\pi[1-\tau^\pi]\left[p\frac{\partial Q}{\partial v} - \left[\frac{\mu\theta^{-\beta}}{\lambda}\phi^L L\right]\right] = 0 \tag{A.48}$$

$$\frac{\partial \Pi}{\partial I} = -\alpha^v\alpha^\pi[1-\tau^\pi]\phi^I + \chi = 0, \tag{A.49}$$

$$\frac{\partial \Pi}{\partial K} = \alpha^v\alpha^\pi[1-\tau^\pi]p\frac{\partial Q}{\partial K} - \alpha^v\alpha^\pi[1-\tau^\pi]\phi^K - \chi\delta = 0. \tag{A.50}$$

That is,

$$p\frac{\partial Q}{\partial v} = \frac{\mu\theta^{-\beta}}{\lambda}\phi^L L, \tag{A.51}$$

$$p\frac{\partial Q}{\partial K} = \phi^K + \delta\phi^I, \tag{A.52}$$

where

$$\frac{\partial Q}{\partial K} = \left[[\psi^Q]^{\frac{1}{\epsilon}}K^{1-\frac{1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}}[eL]^{1-\frac{1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}\left[\frac{\psi^Q}{K}\right]^{\frac{1}{\epsilon}} \tag{A.53}$$

$$\frac{\partial Q}{\partial v} = \left[[\psi^Q]^{\frac{1}{\epsilon}}K^{1-\frac{1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}}[eL]^{1-\frac{1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}\left[\frac{1-\psi^Q}{eL}\right]^{\frac{1}{\epsilon}}\frac{\mu\theta^{-\beta}}{\lambda}L. \tag{A.54}$$

Capital The division of equations (A.52) and (A.51) leads to

$$\left[\frac{\psi^Q eL}{[1-\psi^Q]K}\right]^{\frac{1}{\epsilon}} = \frac{\phi^K + \delta\phi^I}{\phi^L}. \tag{A.55}$$

where the marginal rate of substitution, equals the ratio of factor costs. Equation (A.55) leads to the optimal amount of capital as a function of the employment rate, which is determined in the labor market, such that

$$K = \frac{\psi^Q eL}{1-\psi^Q}\left[\frac{\phi^K(r) + \delta\phi^I(r)}{\phi^L(\bar{w},\theta)}\right]^{-\epsilon}. \tag{A.56}$$

Output After replacing equation A.56 in the production function, it leads to

$$\begin{aligned} Q &= \left[[\psi^Q]^{\frac{1}{\epsilon}}\left[\frac{\psi^Q eL}{1-\psi^K}\left[\frac{\phi^K + \delta\phi^I}{\phi^L}\right]^{-\epsilon}\right]^{\frac{\epsilon-1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}}[eL]^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \frac{eL}{1-\psi^Q}\left[\psi^Q\left[\frac{\phi^K + \delta\phi^I}{\phi^L}\right]^{1-\epsilon} + [1-\psi^Q]\right]^{\frac{\epsilon}{\epsilon-1}}. \end{aligned}$$

Denote

$$\alpha^Q = \frac{\psi^Q[\phi^K + \delta\phi^I]^{1-\epsilon}}{\psi^Q[\phi^K + \delta\phi^I]^{1-\epsilon} + [1-\psi^Q][\phi^L]^{1-\epsilon}}. \tag{A.57}$$

Then, the optimal production is equal to

$$Q = \frac{eL}{1-\psi^Q}\left[\frac{1-\alpha^Q}{1-\psi^Q}\right]^{\frac{\epsilon}{1-\epsilon}}. \tag{A.58}$$

Rents obtained from a vacancy filled The rents obtained from a vacancy filled are measured by the marginal increase in the value of the firm due to an increase of employed individuals, $\partial\Pi/\partial eN$, here denoted by Π^f, and equal to

$$\Pi^f = \alpha^v \alpha^\pi [1-\tau^\pi] \left[p\frac{\partial Q}{\partial eN} - [1+\tau^u]\bar{w}\frac{L}{N} \right]. \tag{A.59}$$

Since

$$\begin{aligned}
\frac{\partial Q}{\partial eN} &= \left[[\psi^Q]^{\frac{1}{\epsilon}} K^{1-\frac{1}{\epsilon}} + [1-\psi^Q]^{\frac{1}{\epsilon}} [eL]^{1-\frac{1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} \left[\frac{1-\psi^Q}{eL} \right]^{\frac{1}{\epsilon}} \frac{L}{N}, \\
&= \left[\frac{eL}{1-\psi^Q} \left[\frac{1-\psi^Q}{1-\alpha^Q} \right]^{\frac{\epsilon}{\epsilon-1}} \right]^{\frac{1}{\epsilon}} \left[\frac{1-\psi^Q}{eL} \right]^{\frac{1}{\epsilon}} \frac{L}{N}, \\
&= \left[\frac{1-\alpha^Q}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} \frac{L}{N},
\end{aligned} \tag{A.60}$$

equation A.59 can be also expressed as

$$\Pi^f = \alpha^v \alpha^\pi [1-\tau^\pi] \left[p \left[\frac{1-\alpha^Q}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} - [1+\tau^u]\bar{w} \right] \frac{L}{N}. \tag{A.61}$$

A.1.2 Labor Market

Let $r^\tau = [1-\alpha^r\tau^r]r$. Solving the system of Bellman equations:

$$\begin{aligned}
V_y^e &= U_y^e + [1+r^\tau]^{-1}[\lambda V_y^u + [1-\lambda]V_y^e], \\
V_y^u &= U_y^u + [1+r^\tau]^{-1}[\mu\theta^{1-\beta} V_y^e + [1-\mu\theta^{1-\beta}]V_y^u], \\
V^f &= \Pi^f + [1+r^\tau]^{-1}[\lambda V^v + [1-\lambda]V^f], \\
V^v &= -c + [1+r^\tau]^{-1}[\mu\theta^{-\beta} V^f + [1-\mu\theta^{-\beta}]V^v],
\end{aligned}$$

leads to

$$V_y^e = \frac{1+r^\tau}{r^\tau}\left[\frac{[r^\tau+\mu\theta^{1-\beta}]U_y^e+\lambda U_y^u}{r^\tau+\mu\theta^{1-\beta}+\lambda}\right], \tag{A.62}$$

$$V_y^u = \frac{1+r^\tau}{r^\tau}\left[\frac{\mu\theta^{1-\beta}U_y^e+[r^\tau+\lambda]U_y^u}{r^\tau+\mu\theta^{1-\beta}+\lambda}\right], \tag{A.63}$$

$$V^f = \frac{1+r^\tau}{r^\tau}\left[\frac{[r^\tau+\mu\theta^{-\beta}]\Pi^f-\lambda c}{r^\tau+\mu\theta^{-\beta}+\lambda}\right], \text{ and} \tag{A.64}$$

$$V^v = \frac{1+r^\tau}{r^\tau}\left[\frac{\mu\theta^{-\beta}\Pi^f-[r^\tau+\lambda]c}{r^\tau+\mu\theta^{-\beta}+\lambda}\right]. \tag{A.65}$$

Job creation

Using the *free entry* condition, $V^v = 0$, in equation A.65, the result determines the equilibrium between rents and costs of filling a vacancy, also known as *job creation equation*, such that

$$\Pi^f = \frac{[r^\tau+\lambda]c}{\mu\theta^{-\beta}}. \tag{A.66}$$

After replacing Π^f with equation A.61, it becomes

$$\alpha^v\alpha^\pi[1-\tau^\pi]\left[p\left[\frac{1-\alpha^Q(\theta)}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}}-[1+\tau^u]\bar{w}\right]\frac{L}{N} = \frac{[r^\tau+\lambda]c}{\mu\theta^{-\beta}}, \text{ or}$$

$$\alpha^v\alpha^\pi[1-\tau^\pi]\left[p\left[\frac{1-\alpha^Q(\theta)}{1-\psi^Q}\right]^{\frac{1}{1-\epsilon}}-[1+\tau^u]\bar{w}\right]eL = \left[1+\frac{r^\tau}{\lambda}\right]cvN$$

Wage determination

Rents and losses from changes in wages

Marginal utility from a change in the wage of an employed individual The marginal utility from a change in the wage of an employed individual is

$$\frac{\partial U_y^e}{\partial w_y} = [C_y^e]^{\frac{1}{\eta}} \frac{\partial C_y^e}{\partial w_y} - \psi^H [H_y^e]^{\frac{1}{\gamma}} \frac{\partial H_y^e}{\partial w_y} = \frac{1}{w_y} \left[[C_y^e]^{1+\frac{1}{\eta}} \Theta_{C,w} - \psi^H [H_y^e]^{1+\frac{1}{\gamma}} \Theta_{H,w} \right]. \quad \text{(A.67)}$$

Equation A.67 is a function of consumption when the individual is employed and hours of labor (equations A.17 and A.20), and wage elasticities of labor and consumption (equations A.26 and A.24).

Marginal change in rents from a filled vacancy due to changes in the average wage The marginal change in rents from a filled vacancy due to changes in the average wage is

$$\frac{\partial \Pi^f}{\partial \bar{w}} = \alpha^v \alpha^\pi [1-\tau^\pi] \frac{L}{N} \left[p \frac{1}{\epsilon - 1} \left[\frac{1-\alpha^Q}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} \left[-\frac{1}{1-\alpha^Q} \right] \left[-\frac{\partial \alpha^Q}{\partial \bar{w}} \right] - [1+\tau^u] \right].$$

Given that

$$\frac{\partial \alpha^Q}{\partial \bar{w}} = -\frac{\psi^Q [\phi^K]^{1-\epsilon} [1-\psi^Q][1-\epsilon][\phi^L]^{-\epsilon} \frac{\partial \phi^L}{\partial \bar{w}}}{[\psi^Q [\phi^K]^{1-\epsilon} + [1-\psi^Q][\phi^L]^{1-\epsilon}]^2} = -\alpha^Q [1-\alpha^Q][1-\epsilon][\phi^L]^{-1} \frac{\partial \phi^L}{\partial \bar{w}}, \quad \text{(A.68)}$$

then,

$$\begin{aligned} \frac{\partial \Pi^f}{\partial \bar{w}} &= \alpha^v \alpha^\pi [1-\tau^\pi] \left[p \frac{1}{\epsilon - 1} \left[\frac{1-\alpha^Q}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} \left[-\frac{1}{1-\alpha^Q} \right] \left[\alpha^Q [1-\alpha^Q][1-\epsilon][\phi^L]^{-1} \frac{\partial \phi^L}{\partial \bar{w}} \right] \frac{L}{N} \right. \\ &\quad \left. - [1+\tau^u] \right], \\ &= \alpha^v \alpha^\pi [1-\tau^\pi] \left[p \left[\frac{1-\alpha^Q}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} \left[\alpha^Q [\phi^L]^{-1} \frac{\partial \phi^L}{\partial \bar{w}} \right] - [1+\tau^u] \right] \frac{L}{N}. \end{aligned}$$

That is,

$$\frac{\partial \Pi^f}{\partial \bar{w}} = \alpha^v \alpha^\pi [1-\tau^\pi][1+\tau^u] \left[p \left[\frac{1-\alpha^Q(\theta)}{1-\psi^Q} \right]^{\frac{1}{1-\epsilon}} \frac{\alpha^Q(\theta)}{\phi^L(\theta)} - 1 \right] \frac{L}{N}. \tag{A.69}$$

The value of α^Q and ϕ^L can be found in equations A.57 and A.47, respectively.

Wage equation The difference between equations A.62 and A.63, and equations A.64 and A.65 are equal to

$$V_y^e - V_y^u = \frac{1+r^\tau}{r^\tau + \mu\theta^{1-\beta} + \lambda}[U_y^e - U_y^u], \tag{A.70}$$

$$V^f - V^v = \frac{1+r^\tau}{r^\tau + \mu\theta^{-\beta} + \lambda}[\Pi^f + c]. \tag{A.71}$$

These differences determine the rents that can be obtained from the accepting the bargained wage, compared with continue being unemployed, or leaving the vacancy still open.

The equilibrium wage is the result of the unconstrained optimization of the Nash bargaining function

$$[V_y^e - V^u]^\sigma [V^f - V^v]^{1-\sigma} \tag{A.72}$$

with respect to w. The first order condition is

$$\sigma \frac{\partial V_y^e}{\partial w_y}[V^f - V^v] + [1-\sigma]\frac{\partial V^f}{\partial \bar{w}}[V_y^e - V_y^u] = 0, \tag{A.73}$$

where

$$\frac{\partial V_y^e}{\partial w_y} = \frac{[1+r^\tau][r^\tau + \mu\theta^{1-\beta}]}{r^\tau[r^\tau + \mu\theta^{1-\beta} + \lambda]}\frac{\partial U_y^e}{\partial w_y}, \text{ and} \tag{A.74}$$

$$\frac{\partial V^f}{\partial \bar{w}} = \frac{[1+r^\tau][r^\tau + \mu\theta^{-\beta}]}{r^\tau[r^\tau + \mu\theta^{-\beta} + \lambda]}\frac{\partial \Pi^f}{\partial \bar{w}} \tag{A.75}$$

Plugging equations A.74 and A.75 in equation A.73, I obtain

$$[1-\sigma]\left[V_y^e - V_y^u\right] = \sigma\left[\frac{\partial U_y^e/\partial w_y}{-\partial \Pi^f/\partial \bar{w}}\right]\left[V^f - V^v\right]. \tag{A.76}$$

Using equations (A.70) and (A.71), I obtain the *wage equation* expressed as the *surplus* obtained from changing to an employed status from being unemployed,

$$U_y^e = U_y^u + \frac{\sigma}{1-\sigma}\left[\frac{\partial U_y^e/\partial w_y}{-\partial \Pi^f/\partial \bar{w}}\right]\left[\Pi^f + c\right] \tag{A.77}$$

Equation A.77 is a function of U_y^e, U_y^u (equations 2.2 and 2.3), $\partial U_y^e/\partial w_y$, $\partial \Pi^f/\partial \bar{w}$ (equations A.67 and A.69), and Π^f (equation A.59).

A.1.3 Government

$$
\begin{aligned}
pG = &\sum_y N_y \left[\tau^C p \left[eC_y^{Pe} + uC_y^{Pu}\right] + \frac{\tau^C p}{[1-\alpha^r\tau^r]r^S}\left[eC_y^{Fe} + uC_y^{Fu}\right]\right] \\
&+ \sum_y N_y \alpha^r \tau^r r^S p^S \left[eS_y^e + uS_y^u\right] \\
&+ \sum_y N_y \left[\alpha^r \tau^r r A_y + \tau^M M_y\right] + \sum_y N_y \bar{\tau}_y \left[eB_y^e + uB_y^u\right] \\
&- \sum_y N_y [1-\alpha^M] My \\
&+ \tau^\pi B^\pi + \tau^{\text{pro}} K + \alpha^x \tau^X X + \alpha^g \tau^G [-Z]
\end{aligned} \tag{A.78}
$$

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